CSU Math 301
Fall 2016

Homework 11

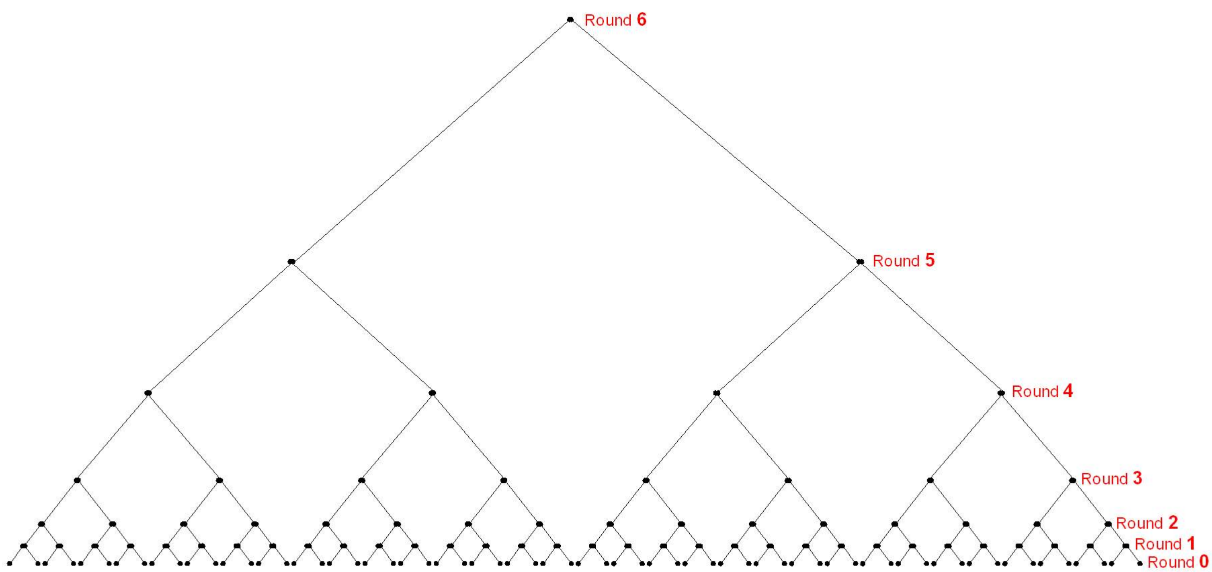
Due Friday, November 18 at the beginning of class
Worth 15 points, instead of the usual 20.

Reading. Sections 8.4, 9.1, 9.2

Remark. Make grammatically correct sentences by adding in just a few English words.

Problems.

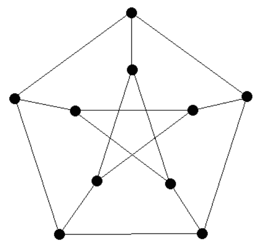
1. The NCAA basketball tournament is a single-elimination tournament with 64 teams; it may be viewed as a rooted binary tree with 64 leaves. An *unfilled NCAA bracket* is a partially labeled binary tree with 64 leaves, where the only labeled vertices are the 64 leaf vertices in Round 0 (each labeled with one of the 64 teams in the tournament). To fill out a bracket you must label the remaining vertices with which team you think will win each game.



(a) Show that the NCAA tournament has 63 games, as follows. Pretend each team brings one new basketball to the tournament. One basketball is used in each match (and then discarded or given away to a lucky fan), and the other still-unused ball is taken by the match's winner to the next round. How many basketballs remain unused at the end of the tournament?

(b) How many ways are there to fill out the bracket? (The labelings on the 64 leaf nodes are fixed; filling out the bracket means choosing a winner for each game.)

2. Show that the Petersen graph drawn below is not planar. You are not allowed to use Kuratowski's Theorem which I mentioned in class. Instead, mimic our proof from class of Theorem 12.2.1. Use the fact (which you don't need to prove) that there are no cycles of length 3 or 4 in the Petersen graph, which then implies $e \geq \frac{5f}{2}$.



3. Show that connecting two vertices u and v in a graph G by a new edge uv creates a new cycle if and only if u and v are in the same connected component of G.