The unreasonably effective interaction between pure and applied mathematics: A case study on persistence images

Henry Adams
Colorado State University

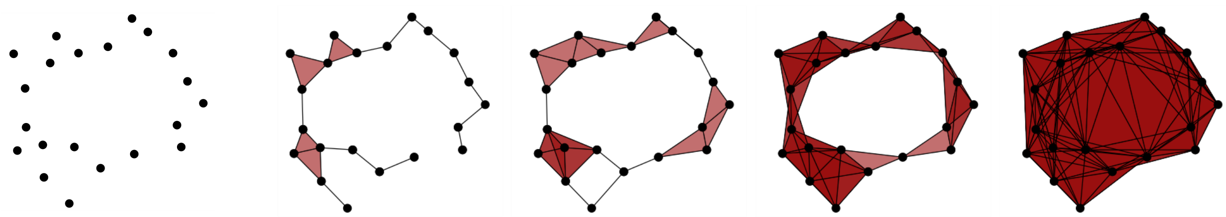
Joint with Sofya Chepushtanova, Tegan Emerson, Eric Hanson, Michael Kirby, Francis Motta, Rachel Neville, Chris Peterson, Patrick Shipman, Lori Ziegelmeier

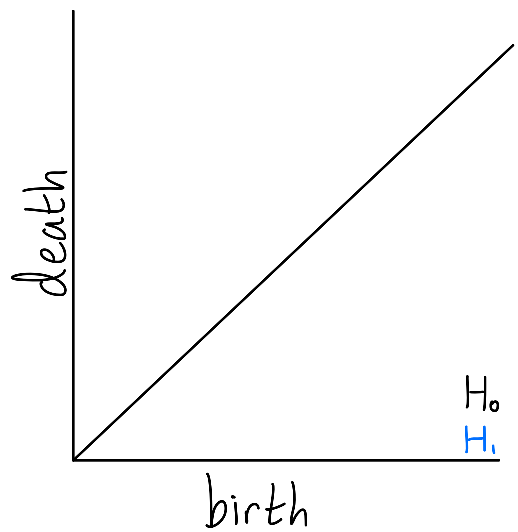
AATRN: 1 or 2 live talks per week
YouTube channel: 2,600 subscribers
20 hours watched per day

The unreasonably effective interaction between pure and applied mathematics: A case study on persistence images

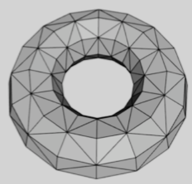
"The unreasonable effectiveness of mathematics in the natural sciences"
By physicist Eugene Wigner, 1960.
The mathematical structure of a physical theory points to improved theory and empirical predictions.

Persistent homology and the shape of data





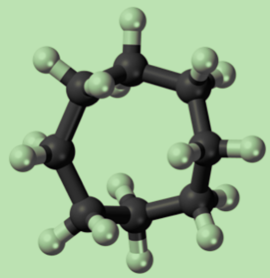
Homology (pure)

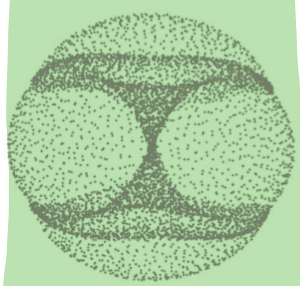


H_0 : rank 1
H_1 : rank 2
H_2 : rank 1

+

Data (applied)





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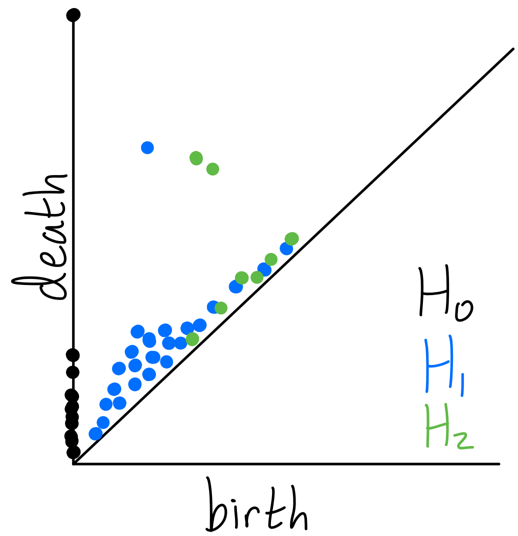
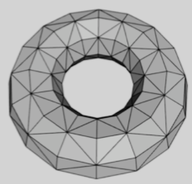
Persistent Homology



Topology of cyclo-octane energy landscape
Martin, Thompson, Coutsias, Watson, 2010

Homology (pure) + Data (applied) = Persistent Homology



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H_1: rank 2
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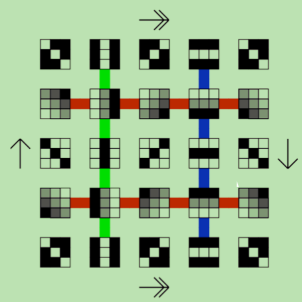
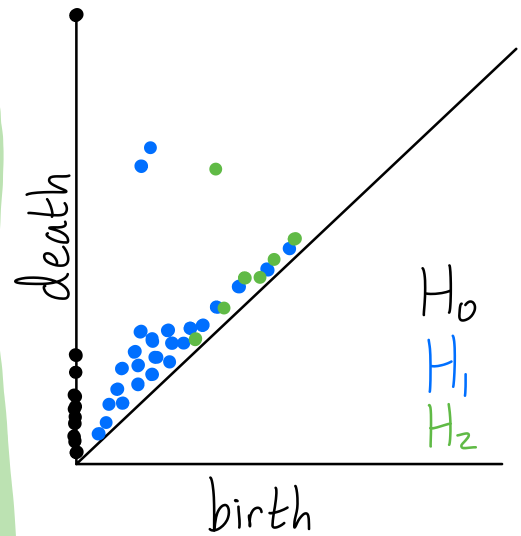


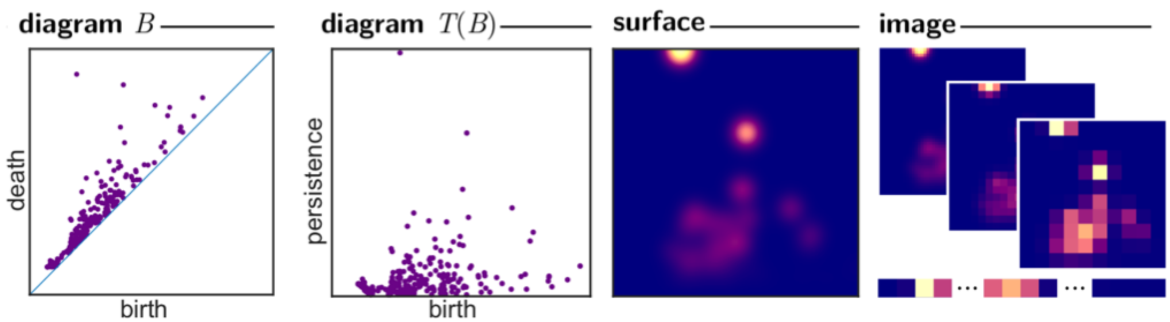


On the local behavior of natural images
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008

Persistent Homology + Machine Learning (applied)

= Persistence Images, Draft 1

(among many many other options)

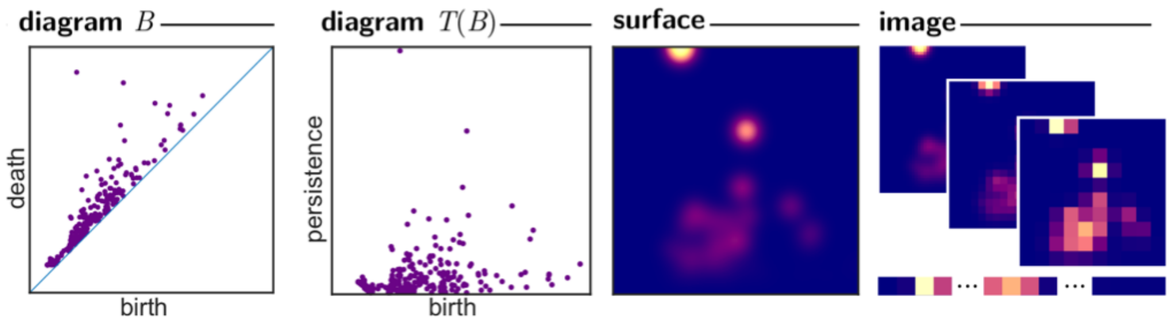


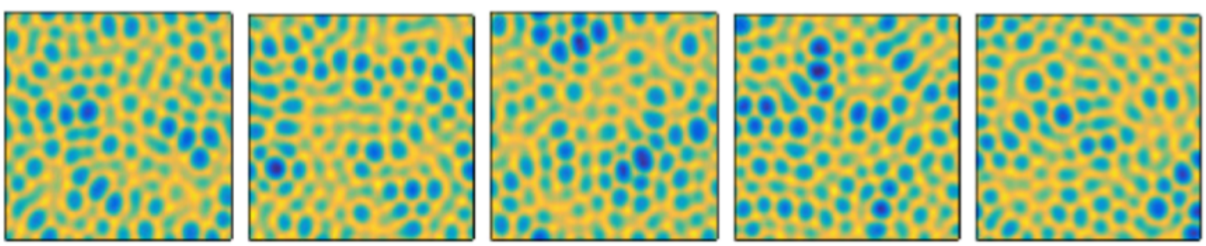
Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017

Persistent Homology + Machine Learning (applied)

= Persistence Images, Draft 1

(among many many other options)



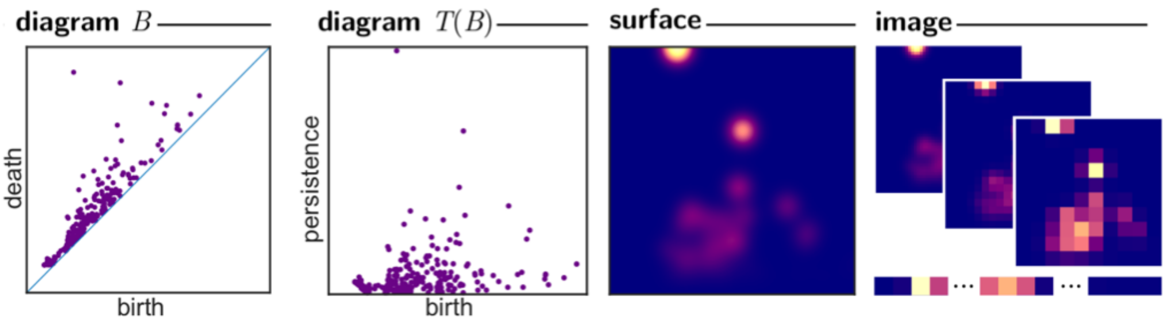


Answer: (from left) $r = 1.75, 2, 1.75, 2, 2.$

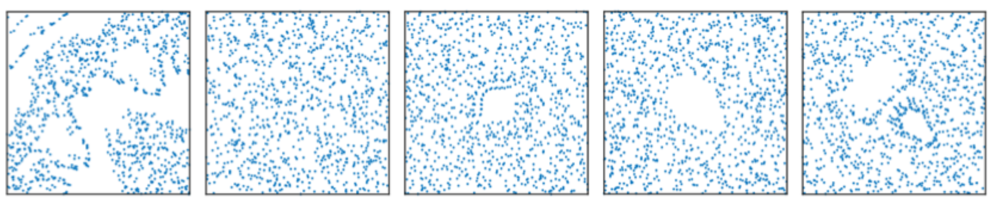
Persistent Homology + Machine Learning (applied)

= Persistence Images, Draft 1

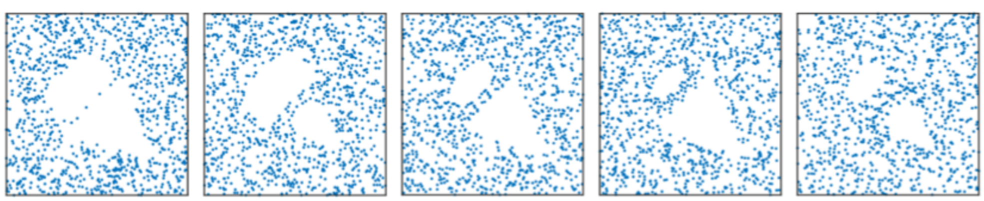
(among many many other options)



Different parameters:

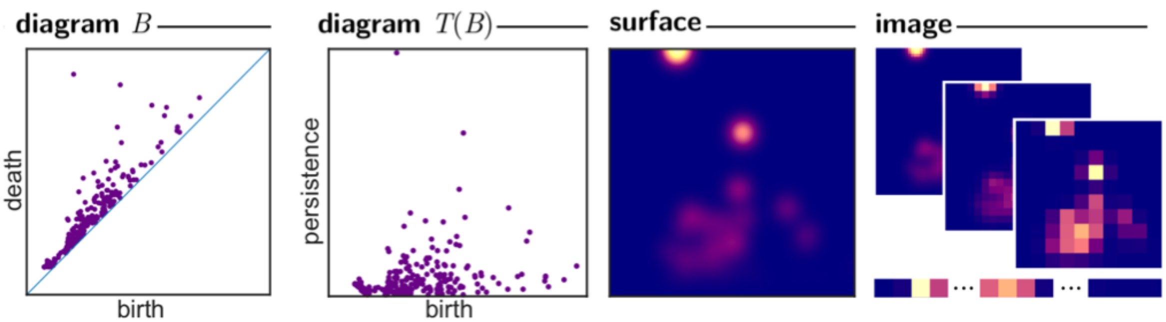


Same parameter:



Persistence Images, Draft 1 + Stability (pure)

= Stable Persistence Images

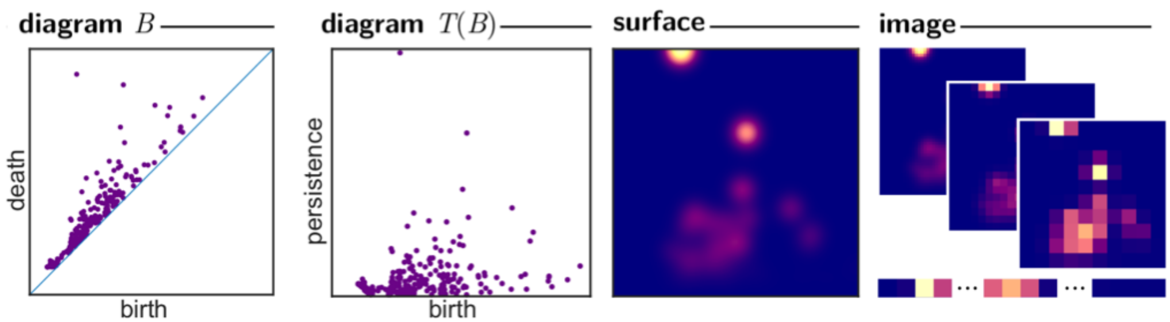


Persistence Images, Draft 1 + Stability (pure)

= Stable Persistence Images

Definition 1 *For B a PD, the corresponding* persistence surface $\rho_B \colon \mathbb{R}^2 \to \mathbb{R}$ *is the function*

$$\rho_B(z) = \sum_{u \in T(B)} f(u)\phi_u(z).$$



Persistence Images, Draft 1 + Stability (pure)

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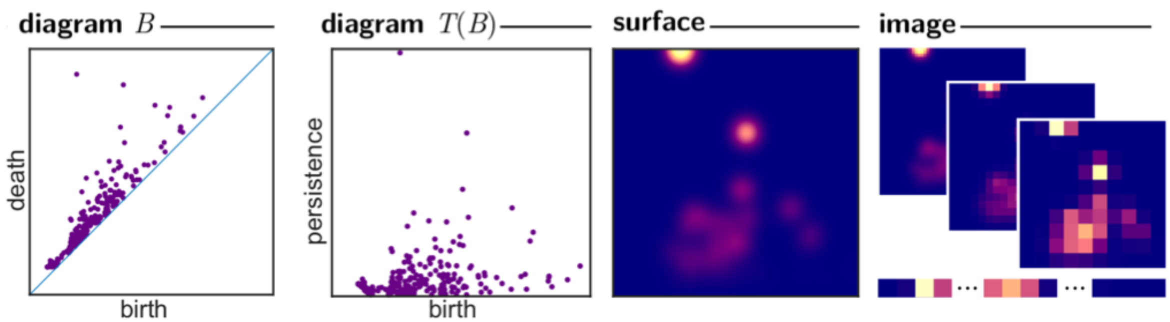

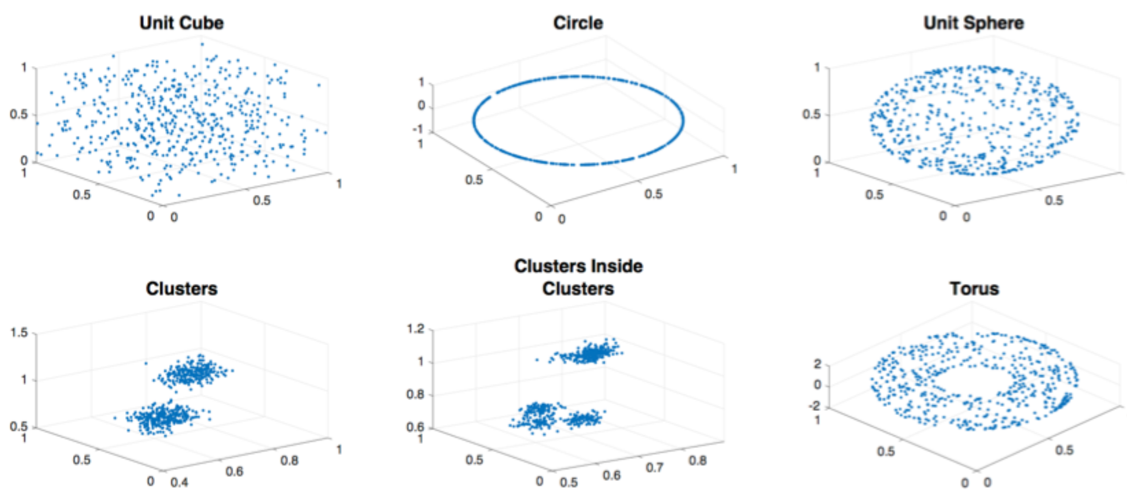

(applied)

Persistence Images, Draft 1 + Stability (pure)

= Stable Persistence Images

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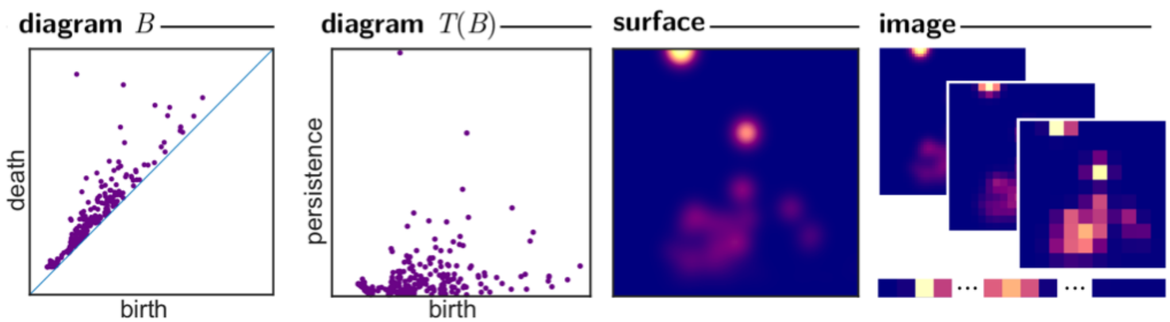


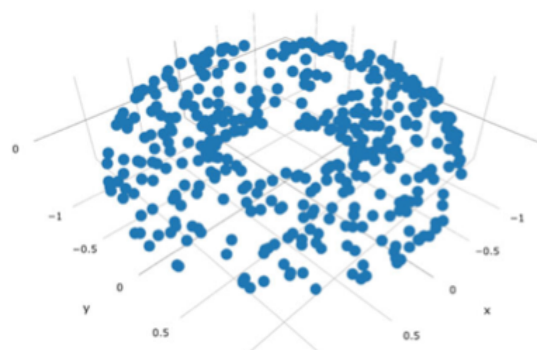
The number of pure and applied questions then exploded!

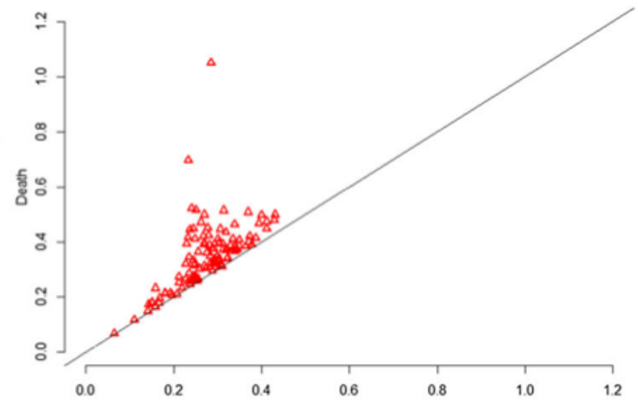
Journal of Applied and Computational Topology (2019) 3:249–283
https://doi.org/10.1007/s41468-019-00032-z

Check for updates

On the choice of weight functions for linear representations of persistence diagrams

Vincent Divol[1,2] · Wolfgang Polonik[3]





Journal of Applied and Computational Topology (2019) 3:249–283
https://doi.org/10.1007/s41468-019-00032-z

On the choice of weight functions for linear representations of persistence diagrams

Vincent Divol[1,2] · Wolfgang Polonik[3]

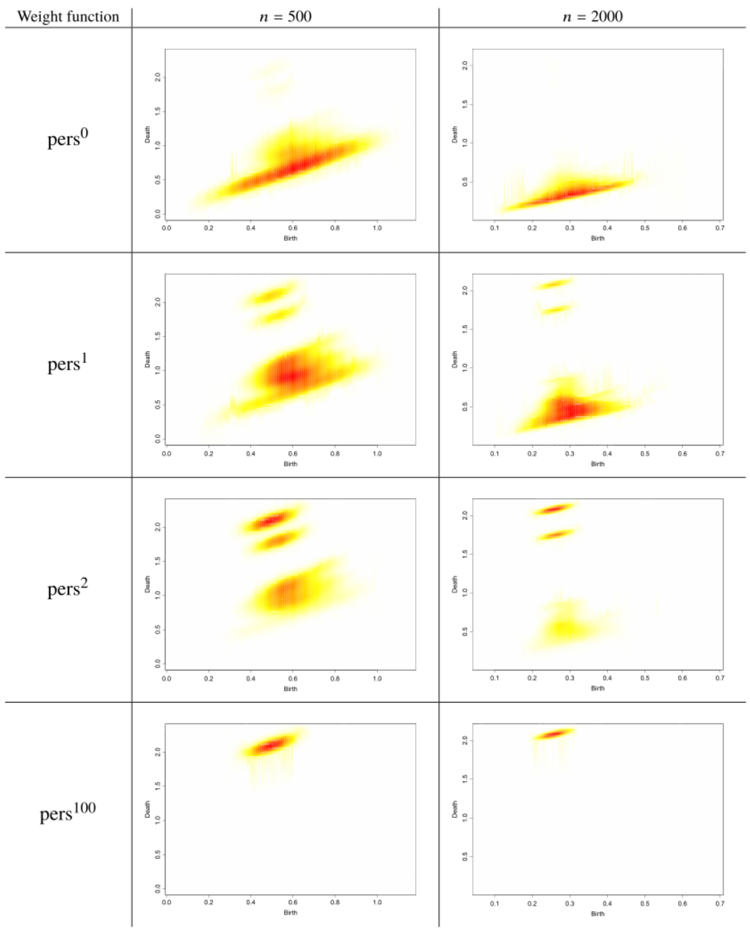


Fig. 2 For $n = 500$ or 2000 points uniformly sampled on the torus, persistence images (Adams et al. 2017) for different weight functions are displayed. For $\alpha < 2$, the mass of the topological noise is far larger than the mass of the true signal, the latter being comprised by the two points with high-persistence. For $\alpha = 2$, the two points with high-persistence are clearly distinguishable. For $\alpha = 100$, the noise has also disappeared, but so has one of the point with high-persistence

J Appl. and Comput. Topology (2018) 1:421–449
https://doi.org/10.1007/s41468-018-0013-5

Persistence diagrams with linear machine learning models

Ippei Obayashi[1] · **Yasuaki Hiraoka**[2,3,4] · **Masao Kimura**[5,6]

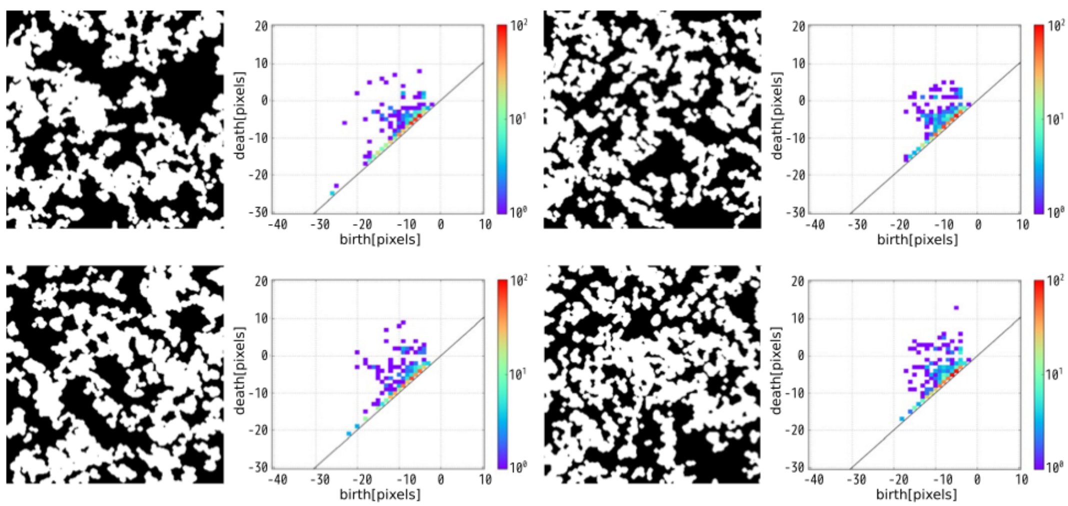


Fig. 4 Input binary images and their 0th persistence diagrams. The left and right two images are sampled from the parameter pairs (**A**) and (**B**), respectively

J Appl. and Comput. Topology (2018) 1:421–449
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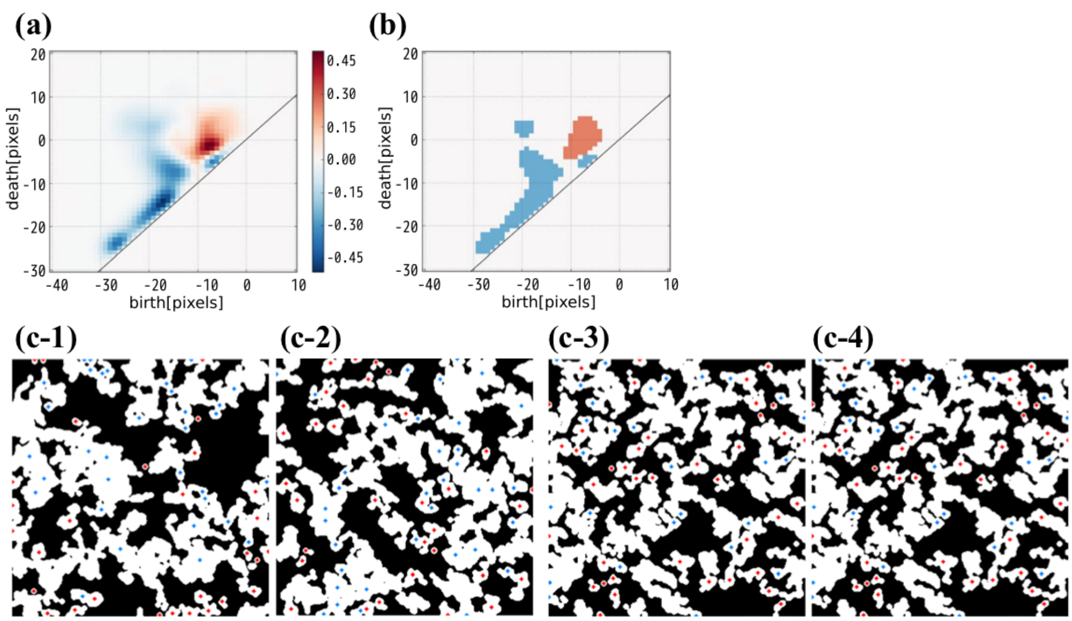


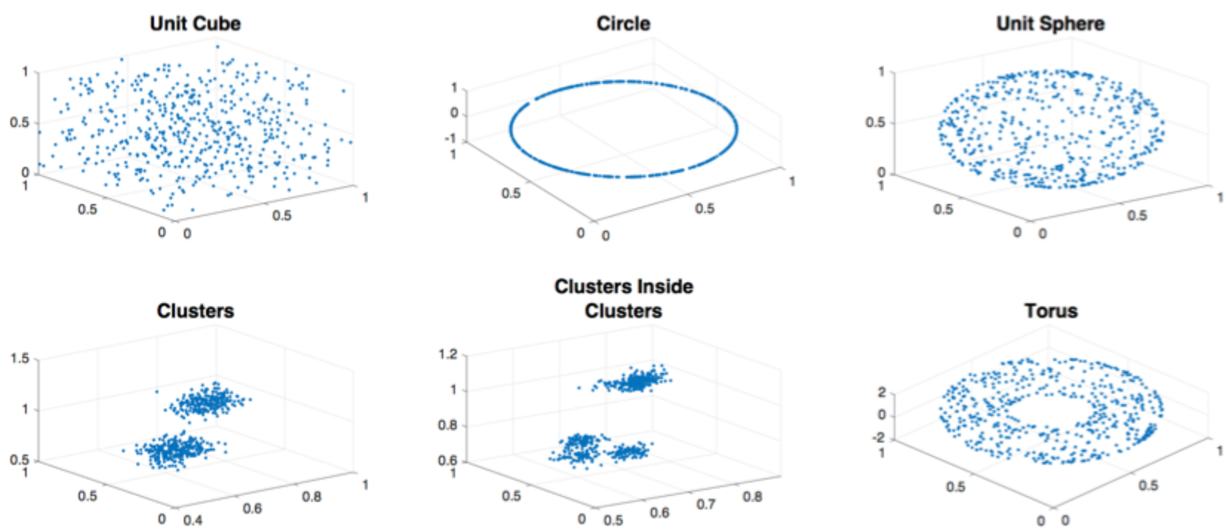
Fig. 5 **a** The reconstructed persistence diagram from the learned vector w. The blue (resp. red) area contributes to the class 0 (resp. 1). **b** A thresholding of (**a**). **c** 1–4 The birth positions of the generators in blue and red areas in (**b**) are plotted with the same color (color figure online)

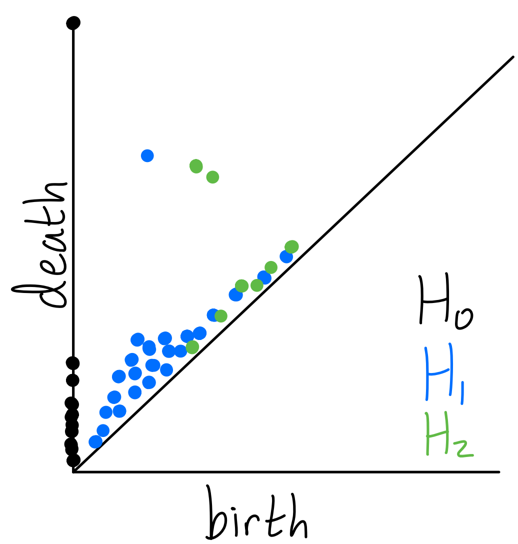
frontiers in Artificial Intelligence

REVIEW
published: 14 May 2021
doi: 10.3389/frai.2021.668302

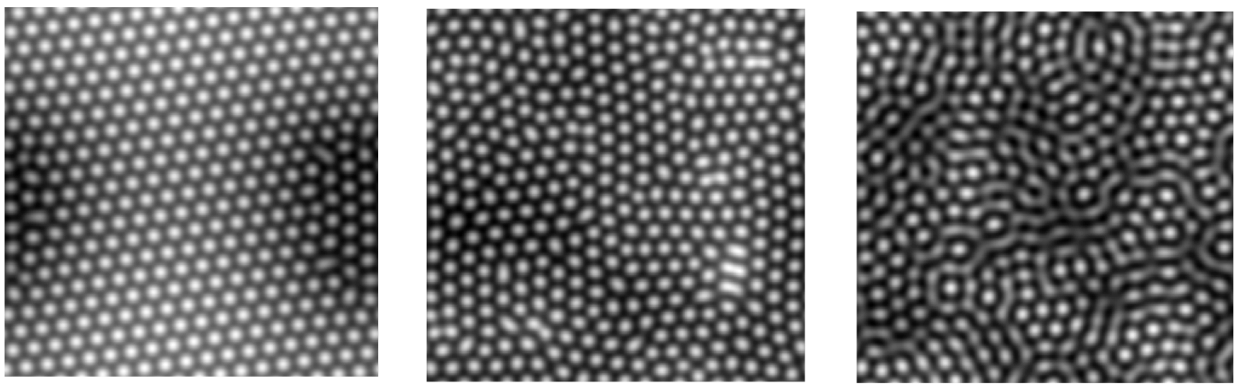
Topology Applied to Machine Learning: From Global to Local

*Henry Adams and Michael Moy**



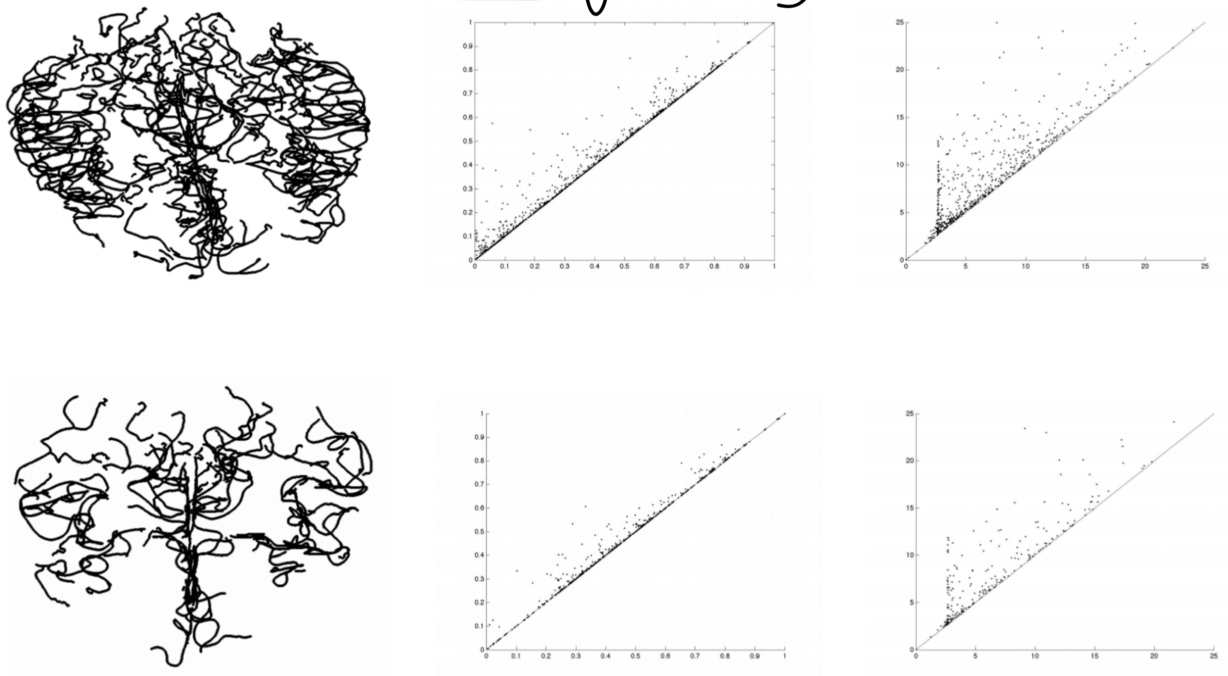


Local geometry



Measures of order for nearly hexagonal lattices
Motta, Neville, Shipman, Pearson, Bradley, 2018

Local geometry



Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skwerer, 2014

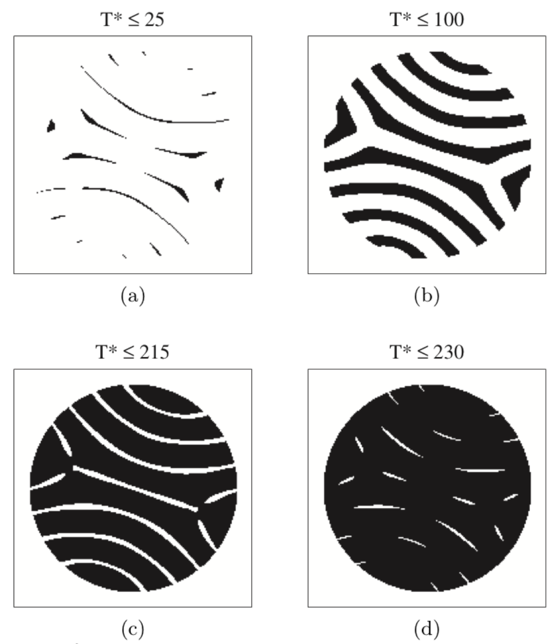
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
Topaz, Ziegelmeier, Halverson, 2015

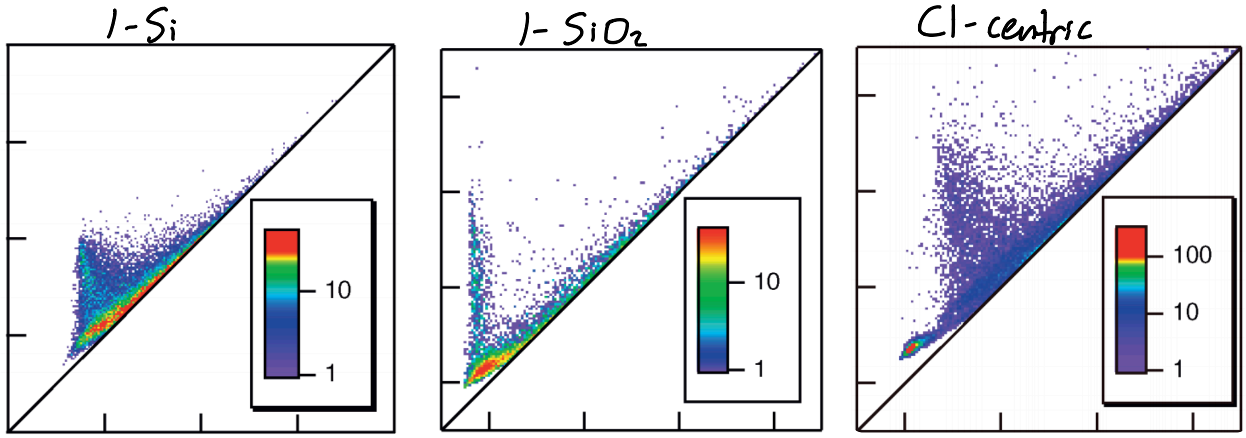
Local geometry



Analysis of Kolmogorov flow and Rayleigh-Bénard convection using persistent homology

Kramár, Levanger, Tithof, Suri, Xu, Paul, Schatz, Mischaikow, 2016

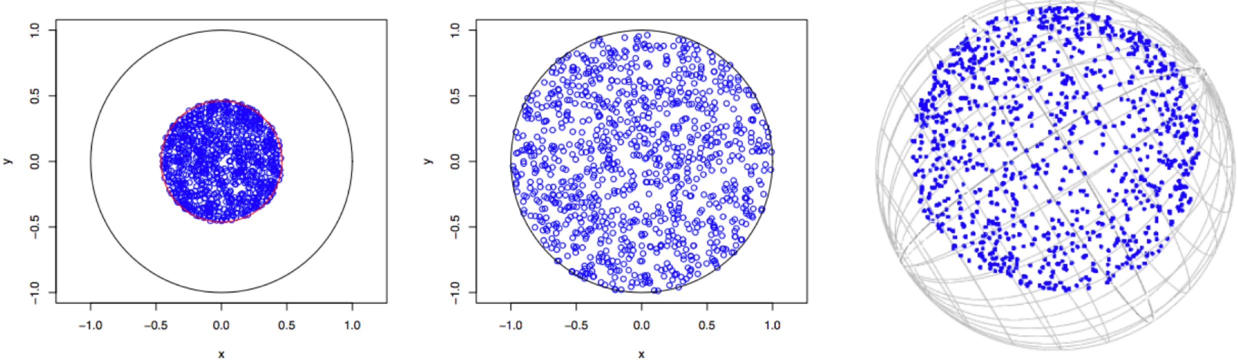
Local geometry



Understanding diffusion patterns of glassy, liquid and amorphous materials via persistent homology analysis
Onodera, Kohara, Tahara, Masuno, Inoue, Shiga, Hirata, Tsuchiya, Hiraoka, Obayashi, Ohara, Mizuno, Sakata, 2019

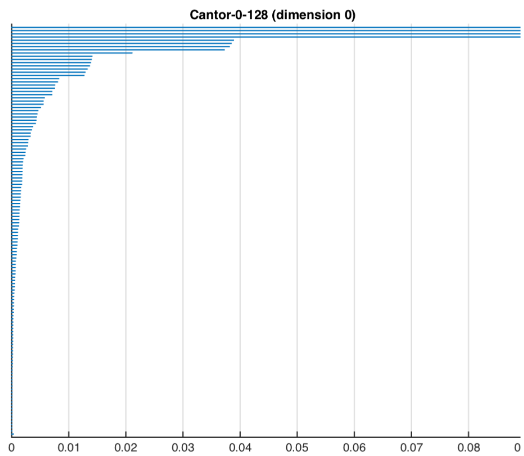
Local geometry

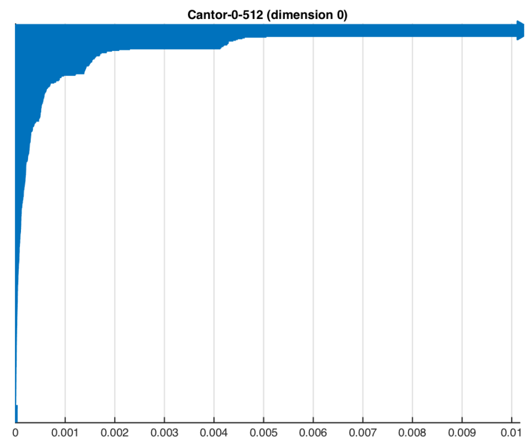
Hyperbolic disk Flat disk Disk on sphere



Persistent homology detects curvature
Bubenik, Hull, Patel, Whittle, 2019

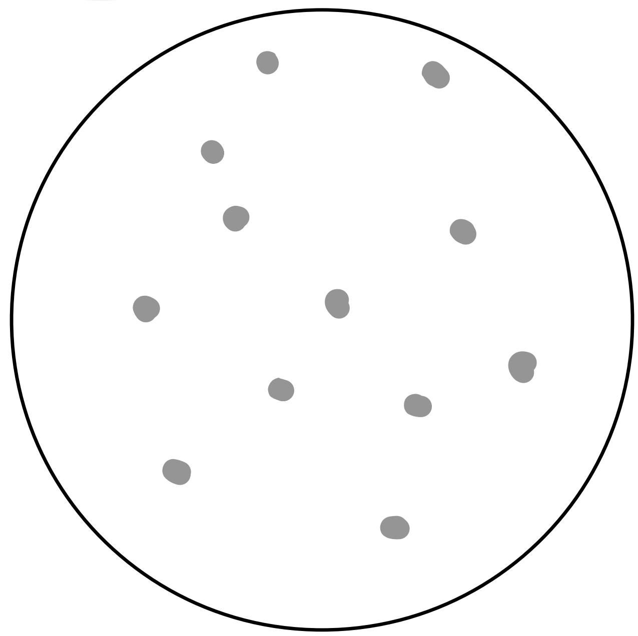
Local geometry





A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020

Local geometry



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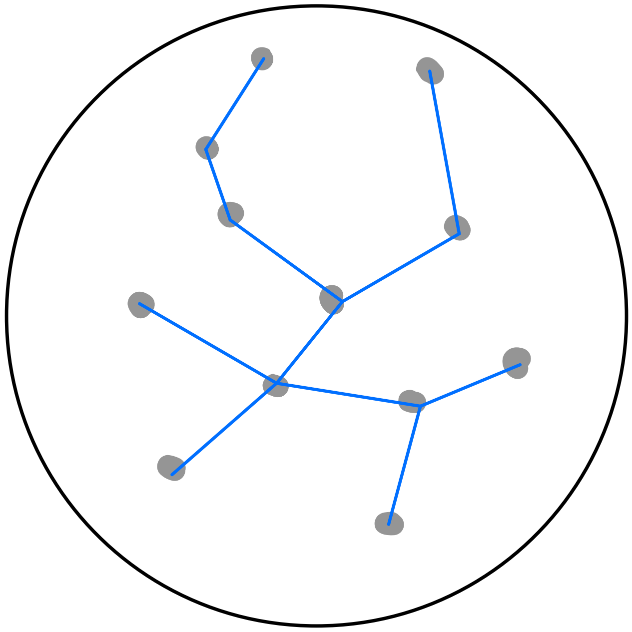

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Adams, Aminian, Farnell, Kirby, Peterson, Mirth, Neville, Shonkwiler, 2020

Further progress in a series of papers by Benjamin Schweinhart

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Thanks!