Mobile Sensors and Pursuit-Evasion: Can Directed Algebraic Topology Help?

Henry Adams, Stanford Mathematics

Geometric and Topological Methods in Computer Science, Aalborg University, January 2010

Introduction

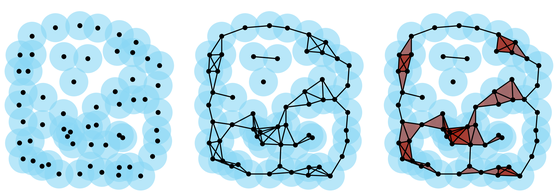
This poster describes an interesting problem.

Applied Setting

This is roughly the set-up of Section 11 of *Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology* by Vin de Silva and Robert Ghrist [1].

Sensors and evaders move continuously in a bounded simply-connected domain $\mathcal{D} \subset \mathbb{R}^2$ during the time interval $t \in [0,1]$. Each sensor covers a unit ball about its center. Let $\mathcal{U}_t \subset \mathcal{D}$ be the covered region at time t. Except for a cycle of immobile sensors which cover the boundary of the domain, $\partial\mathcal{D}$, we do not know the sensor locations. Instead, for all time we know the abstract *communication graph* of the sensors:

- The vertices are the sensors.
- An edge exists if its two sensors are within distance $\sqrt{3}$.



Sensors, their communication graph, and their Rips complex.

Note: the constant $\sqrt{3}$ is chosen so that a triangle in the communication graph corresponds to three overlapping sensors with no gap in the middle.

We would like to use this coordinate-free information to determine: is there an *evasion path* $p : [0,1] \to \mathcal{D}$ with $p(t) \notin \mathcal{U}_t$?

Pure Setting

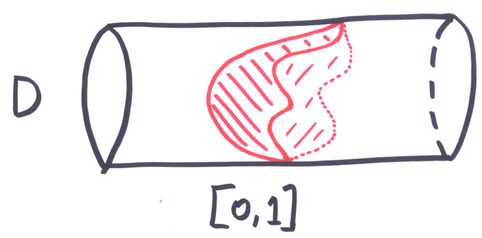
The space-cross-time region $\mathcal{D} \times [0,1]$ has a time-induced partial order.

$$(x,t) \preceq (x',t') \iff t \leq t'$$

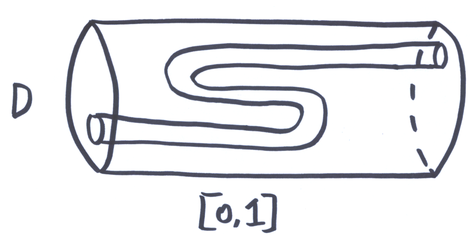
Let $\mathcal{U} \subset \mathcal{D} \times [0,1]$ be the region covered by the sensors. What information about $\mathcal{U}$ does one need in order to determine if there is a directed evasion path in its complement $\mathcal{D} \times [0,1] \setminus \mathcal{U}$? Are there ideas, invariants, or tools from directed algebraic topology which could be helpful?

Can the Criterion of [1] be Sharpened?

The main idea of Theorem 7 of [1] is that if there exists a relative 2-cycle in $H_2(\mathcal{U}, \partial\mathcal{D} \times [0,1])$ whose boundary wraps nontrivially around $\partial\mathcal{D} \times [0,1]$, then no evasion path exists. The actual statement uses Rips complexes instead of $\mathcal{U}$, providing a computable criterion.



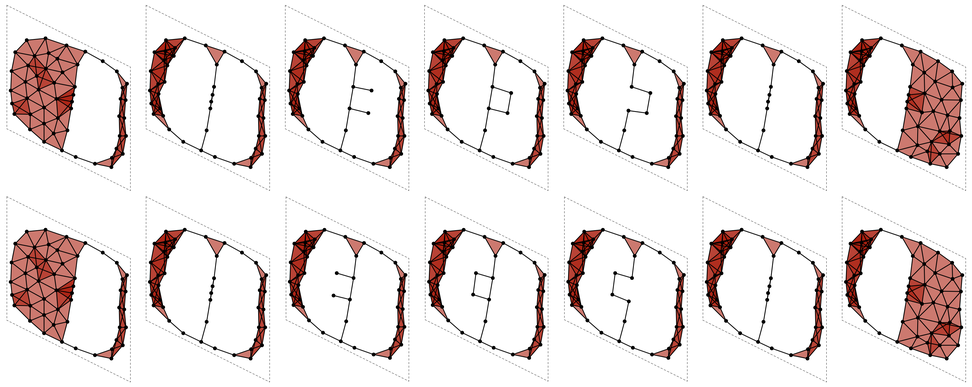
A relative 2-cycle ⇒ no path.



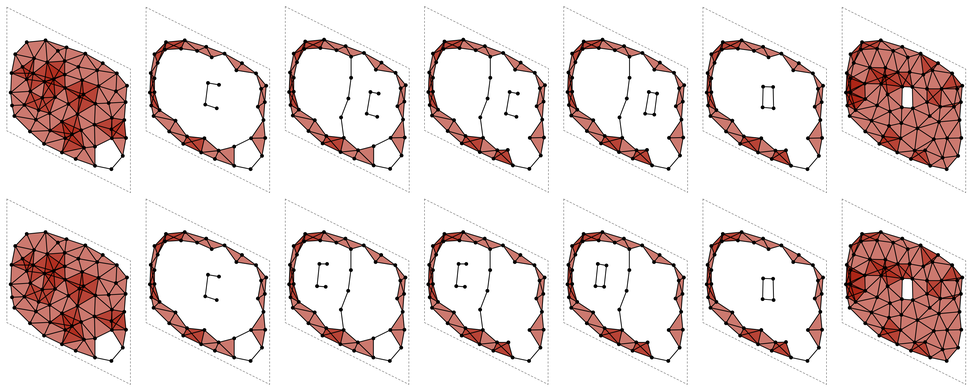
Unfortunately, a physically impossible undirected path in the complement of $\mathcal{U}$ ⇒ no relative 2-cycle.

Dependence on the Embedding

Below are two pairs of sensor networks, drawn as snapshots with time increasing from left to right. The networks in each pair have the same communication graphs for all times. There is also a directed homeomorphism, which acts as the identity on the time coordinate, between the shadows of the Rips complexes in $\mathcal{D} \times [0,1]$. However, the first sensor network in each pair has an evasion path while the second does not.



Pair A: Top row contains evasion path; bottom does not.



Pair B: Top row contains evasion path; bottom does not.

What minimal sensor capabilites might one add to distinguish these examples? Each covered region $\mathcal{U}_t$ is homotopic to a graph, and the embedding type of a possibly disconnected planar graph in $\mathbb{R}^2$ is determined by the cyclic order of the edges around each vertex, the external boundary of each connected component, and the void containing each component. In Pair A, one could identify evasion paths if each sensor knew the cyclic order of its neighbors, a plausible assumption. In Pair B, one would like to track the void containing each connected component. This may require significantly smarter sensors.

Thanks

I would like to thank Robert Ghrist for sharing this problem with me, and Martin Raussen, Lisbeth Fajstrup, and Amra Ibrisevic for organizing the workshop.

References

[1] V. de Silva and R. Ghrist, *Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology*, Int. J. Rob. Res. **25** (2006), no. 12, 1205-1222.