Support Vector Machines and Radon's Theorem

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Colorado State University
Joint with Brittany Story, Elin Farnell

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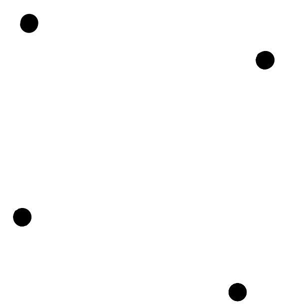
<u>Radon's Theorem</u> Given $n+2$ points in $\mathbb{R}^n$, there exist two disjoint subsets whose convex hulls intersect.



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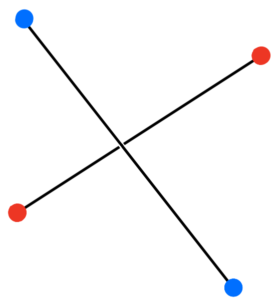

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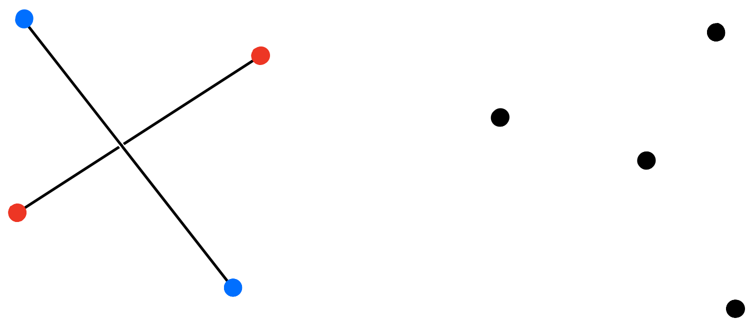

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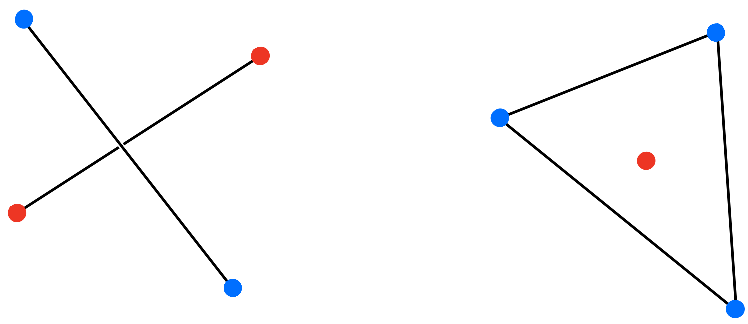

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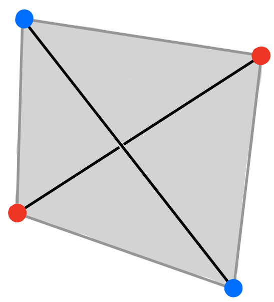

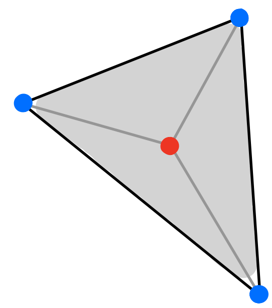

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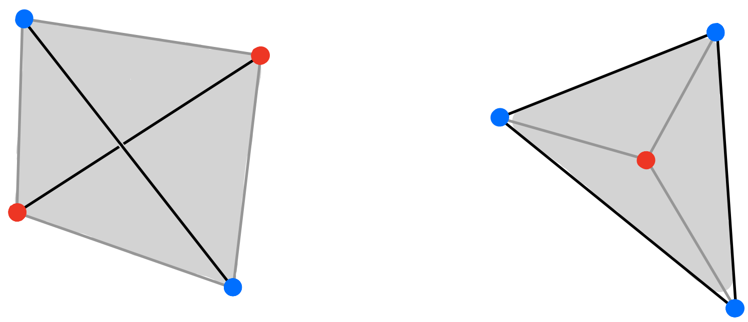

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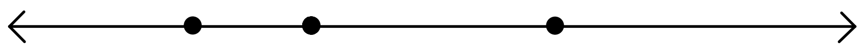

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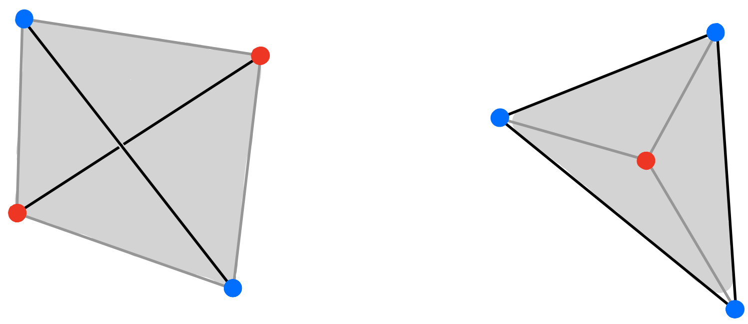

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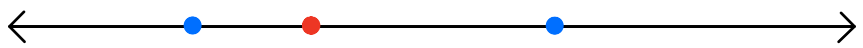

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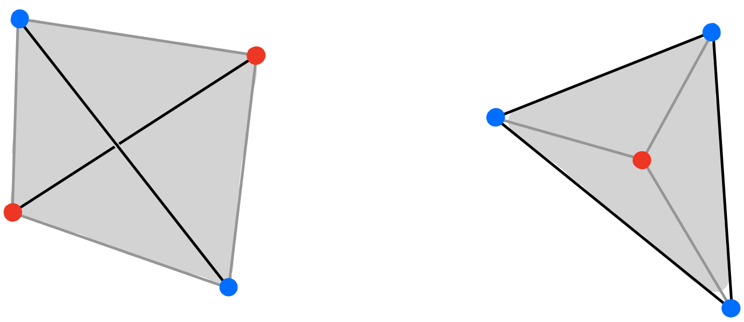

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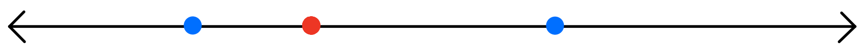
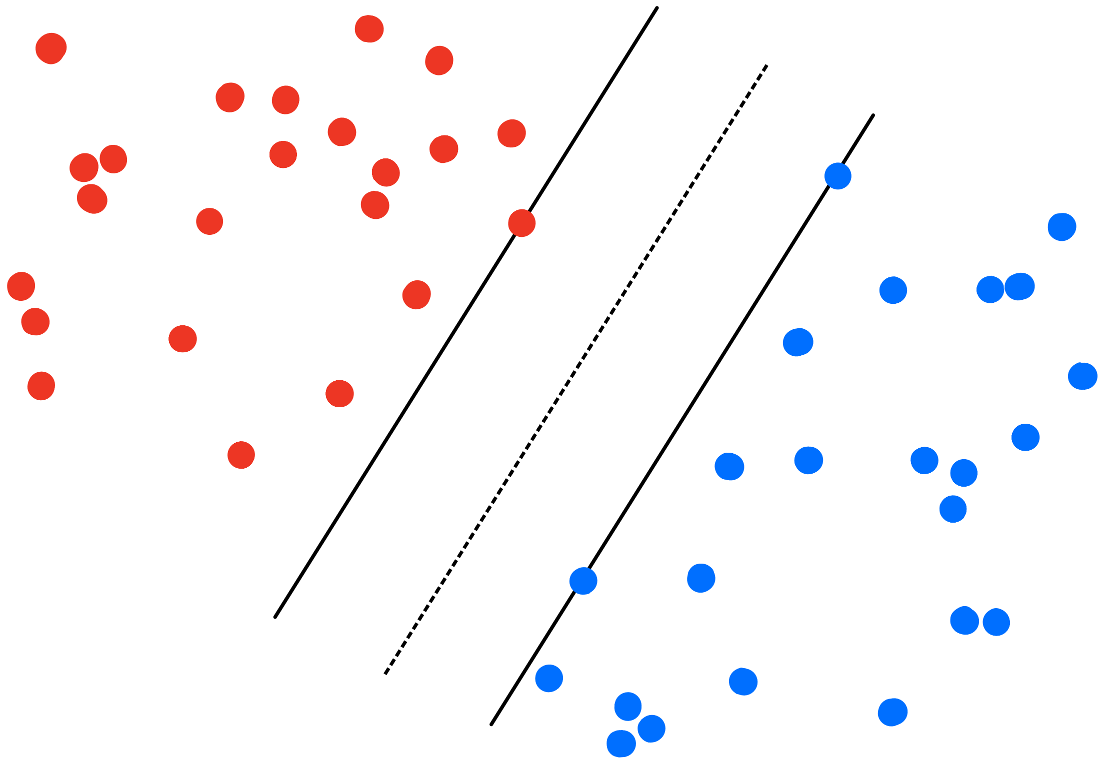


$\mathbb{R}^0$:

Support Vector Machine (SVM)

A support vector machine finds the hyperplane separating two classes of data with the widest margin.



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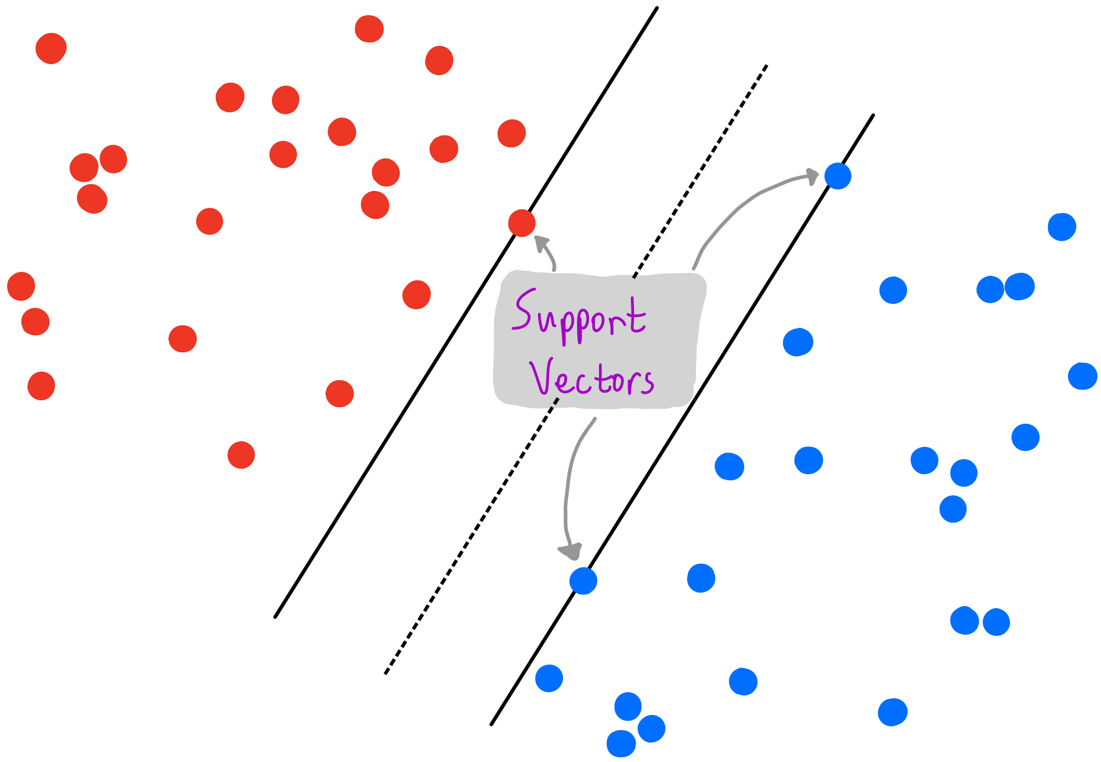

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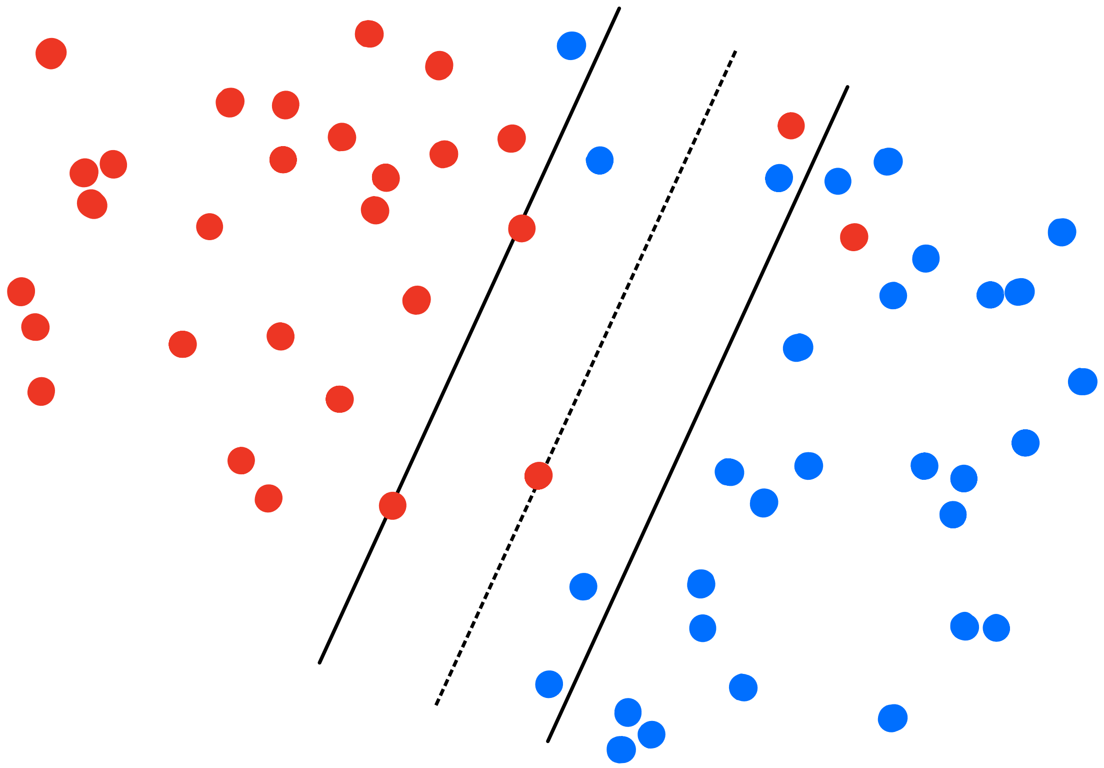

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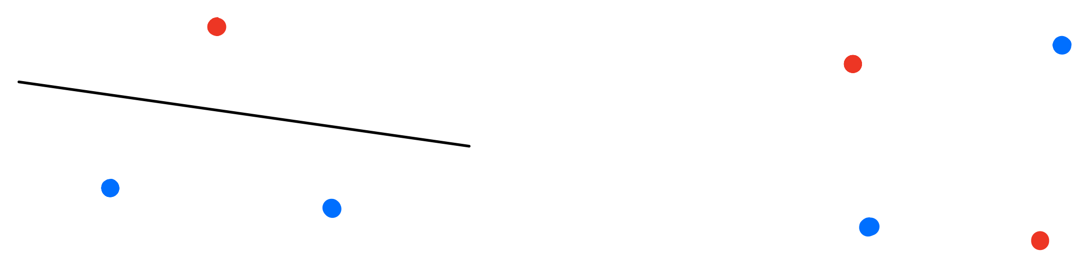

The classical application of Radon's theorem to SVMs is to show that the Vapnik-Chervonenkis (VC) dimension of affine separators in $\mathbb{R}^n$ is $n+1$.

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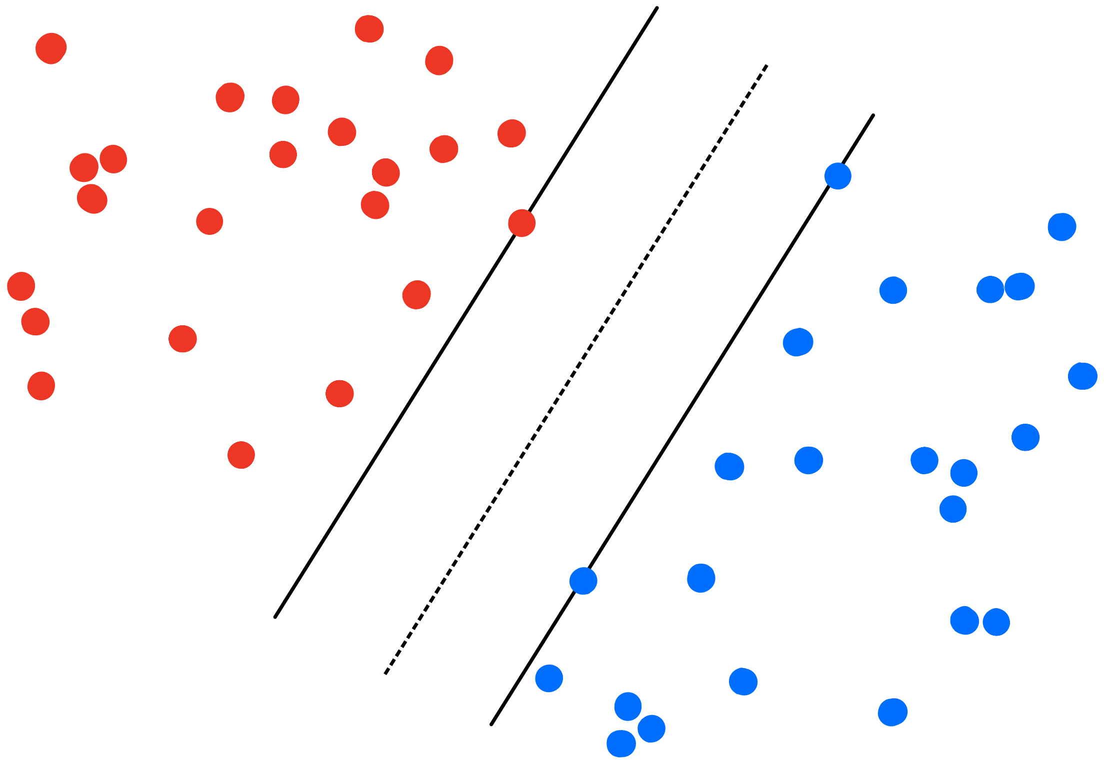

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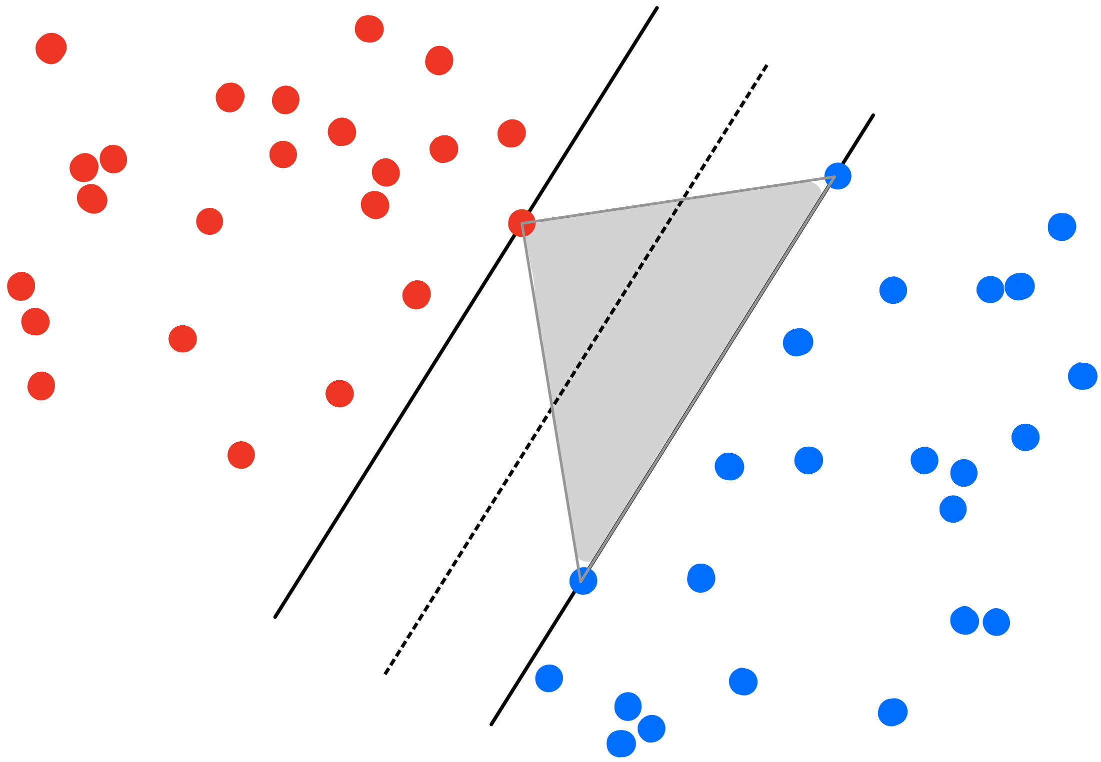

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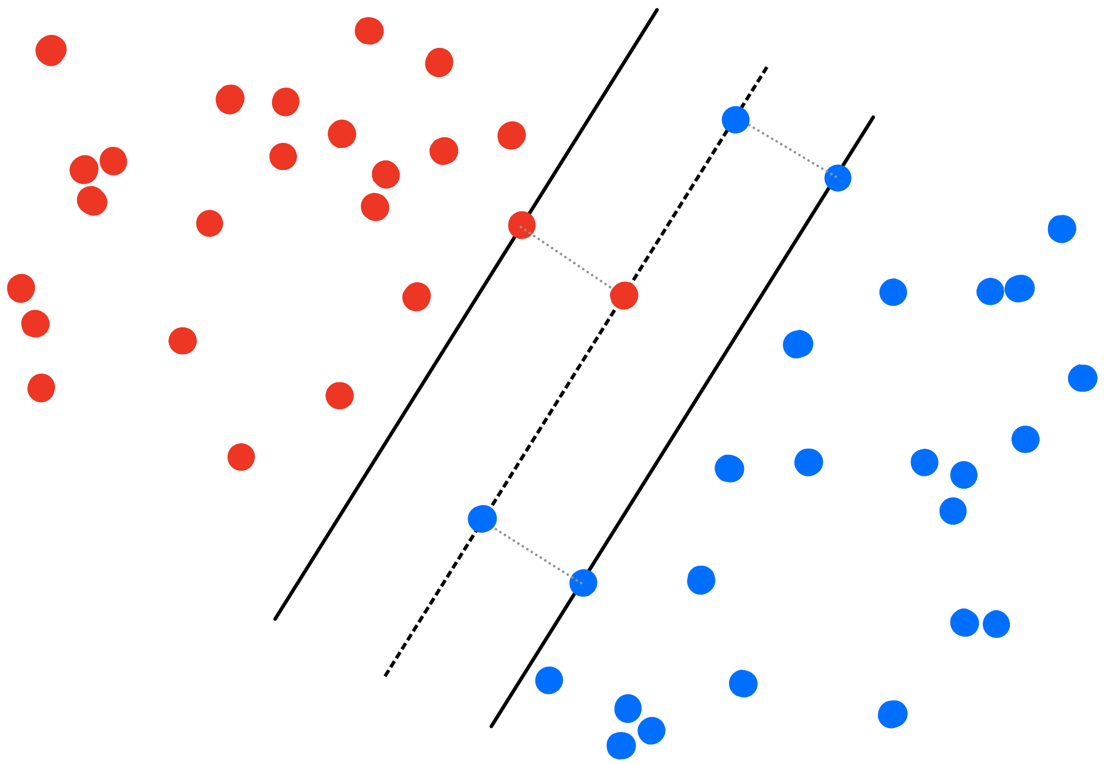

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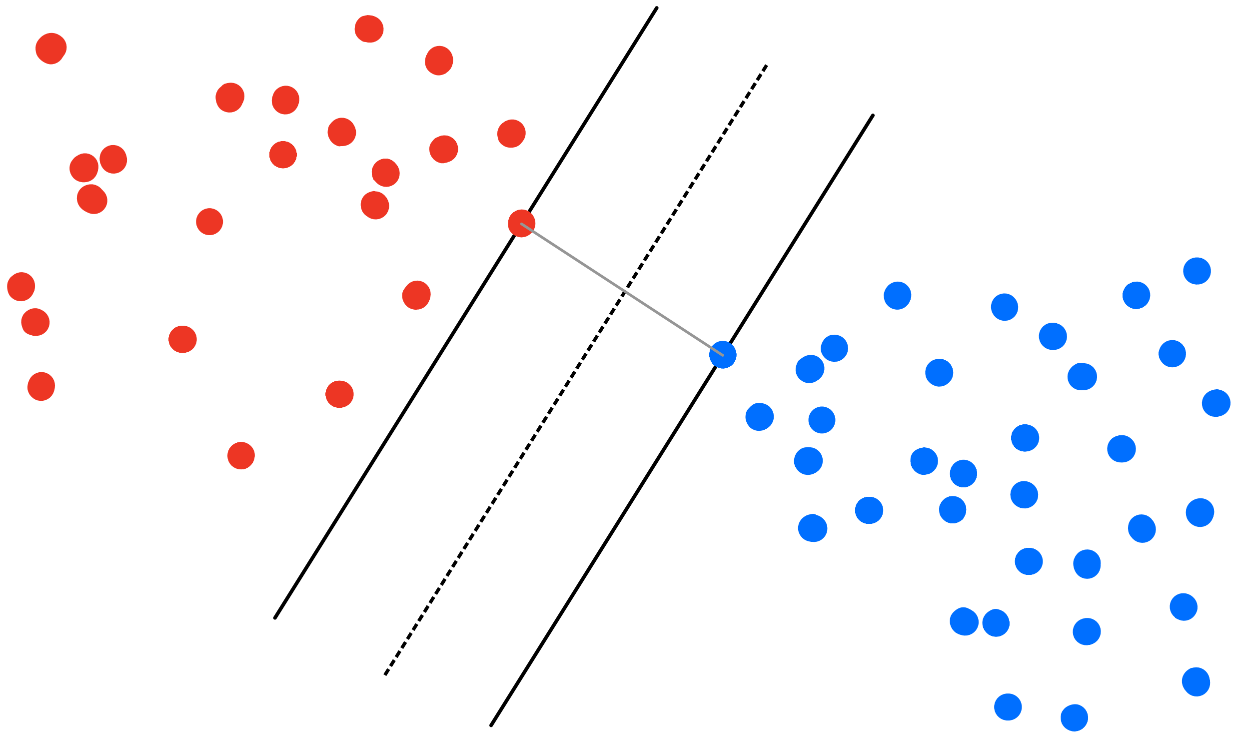

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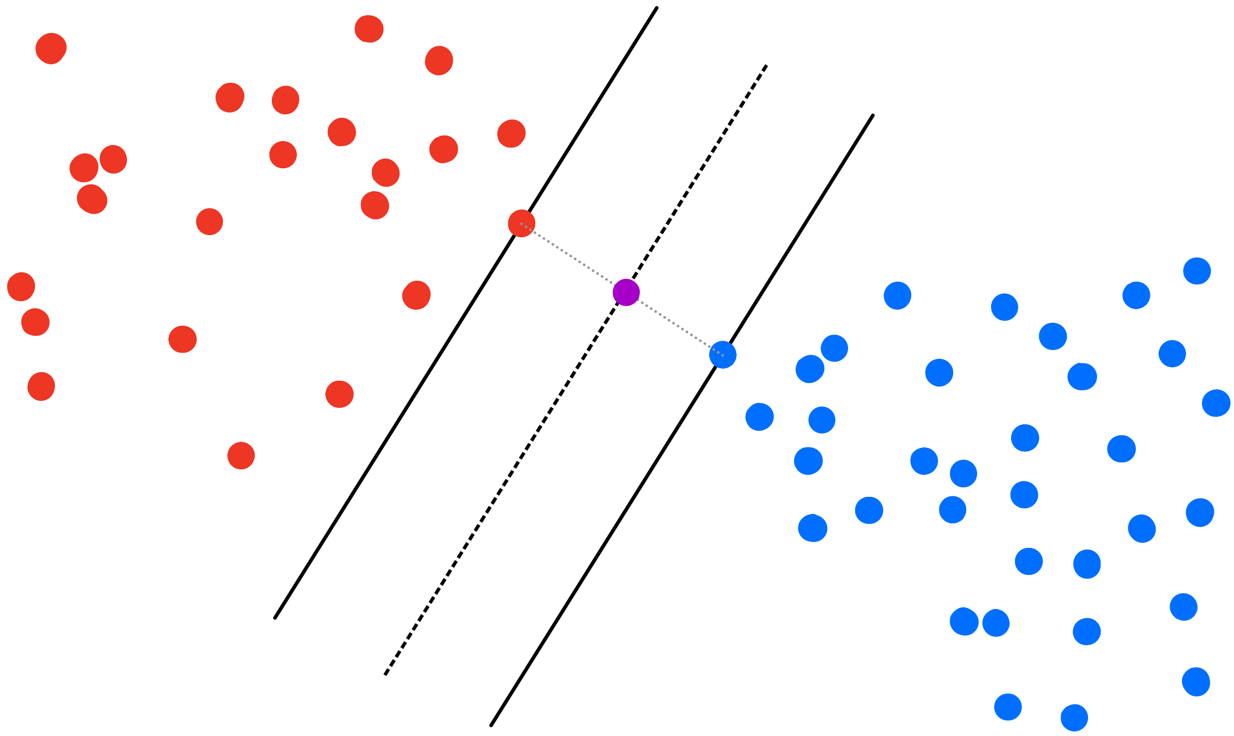

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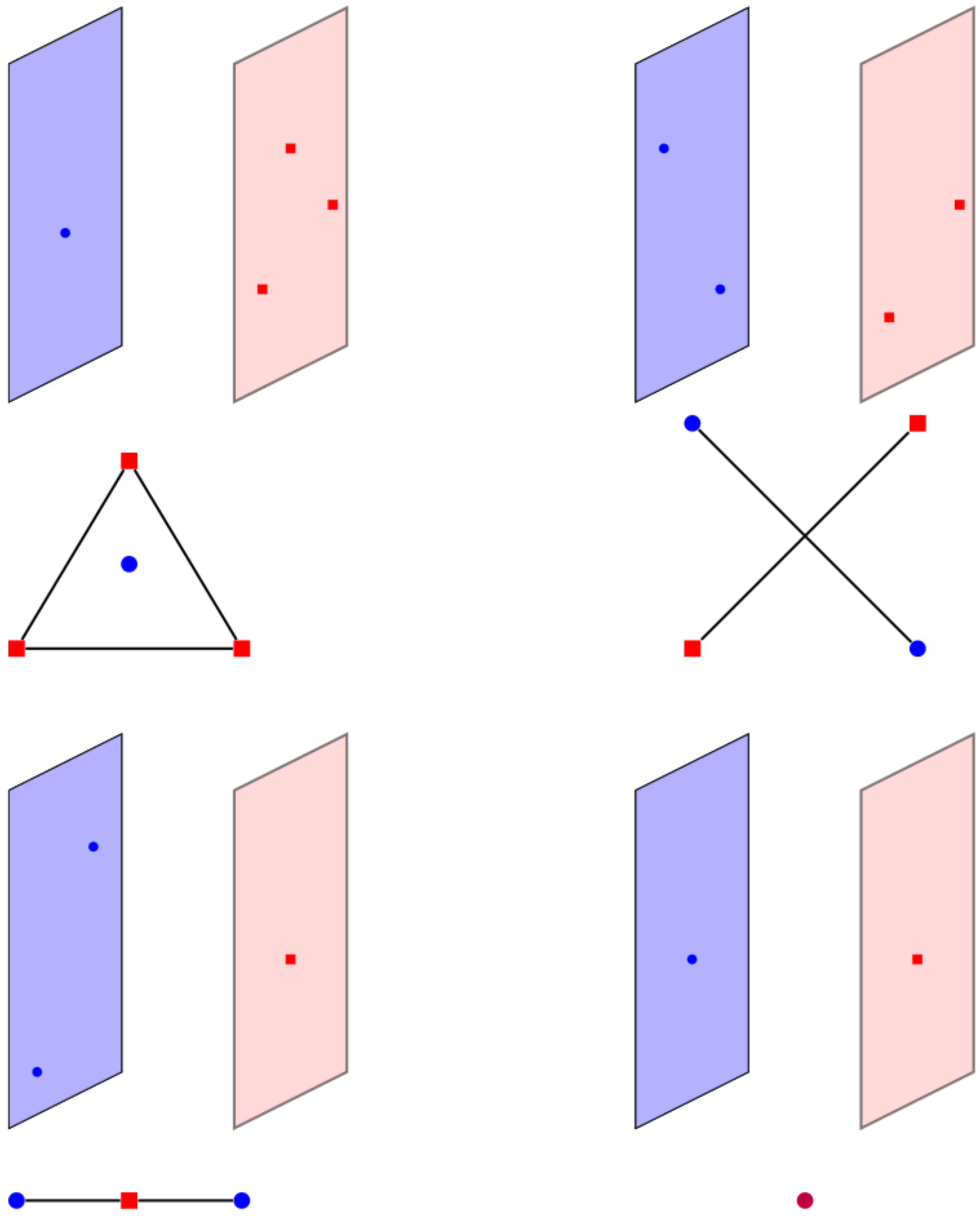

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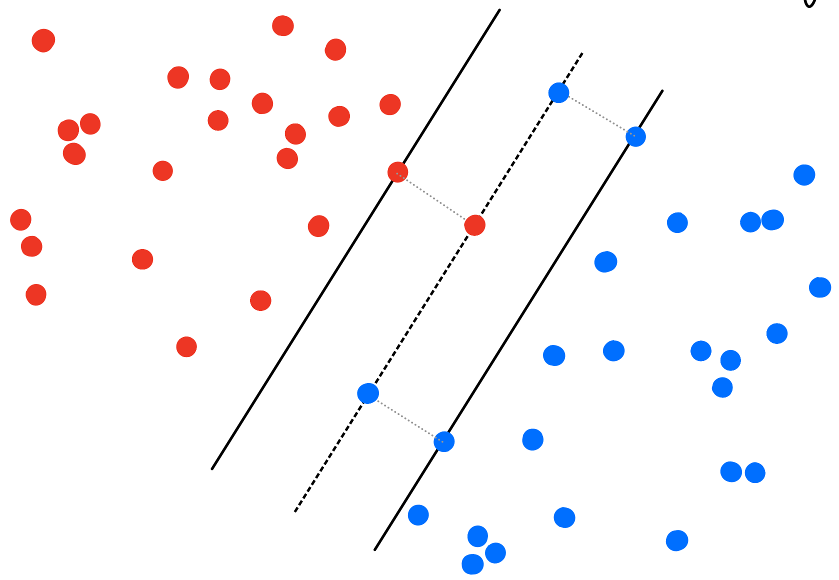

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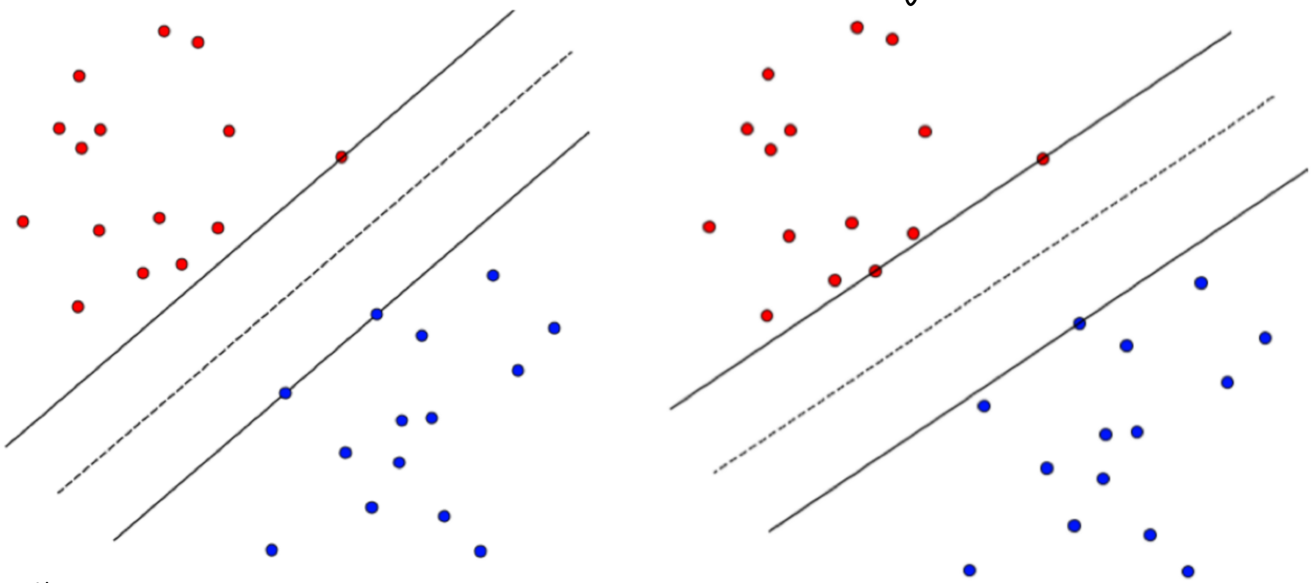

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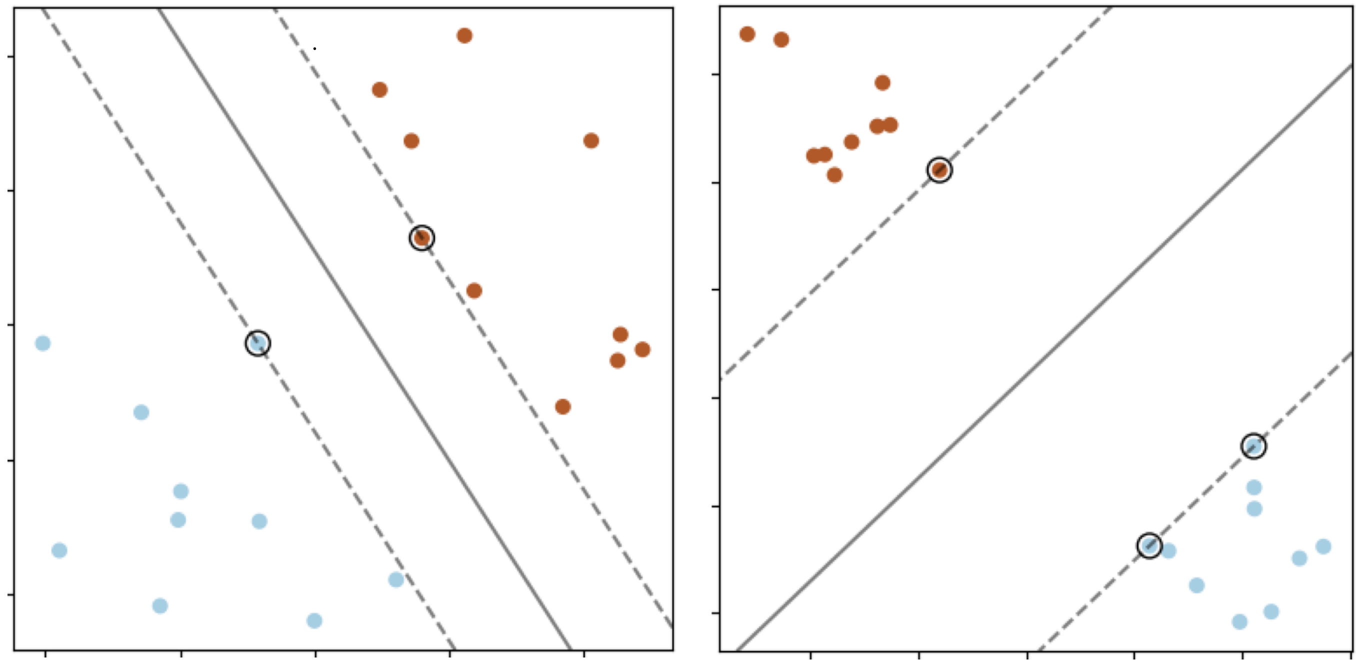

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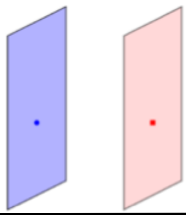
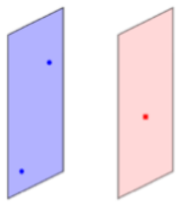
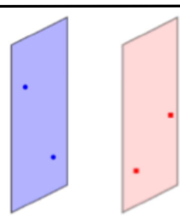
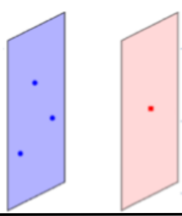
1,000 trials in $\mathbb{R}^3$

Support vectors: 1 / 1 	554
Support vectors: 2 / 1 	367
Support vectors: 2 / 2 	47
Support vectors: 3 / 1 	32

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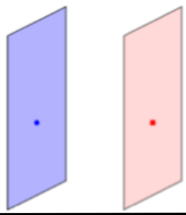
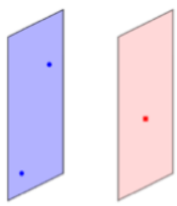
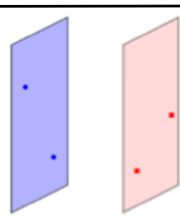
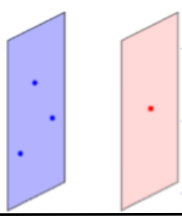
Distance between classes	Small	Medium	Large
Support vectors: 1/1 	279	554	758
Support vectors: 2/1 	458	367	221
Support vectors: 2/2 	145	47	9
Support vectors: 3/1 	118	32	12

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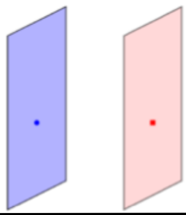
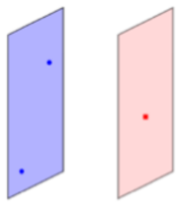
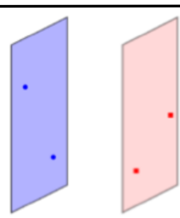
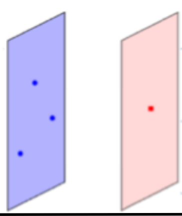
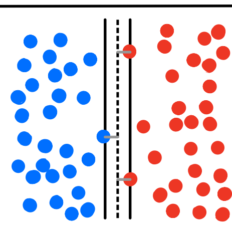
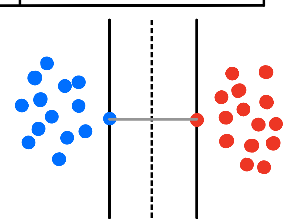
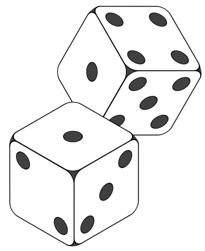
Support vectors: 1/1 	279	554	758
Support vectors: 2/1 	458	367	221
Support vectors: 2/2 	145	47	9
Support vectors: 3/1 	118	32	12



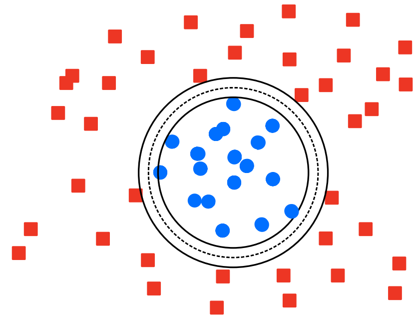


Questions

Firey dice problem?



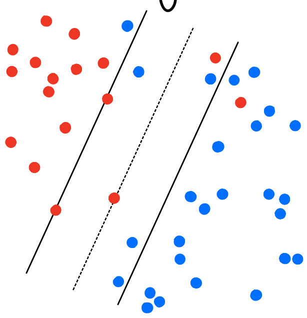
Kernel SVM?



Spherical or ellipsoidal SVM?

Tverberg's theorem and multiclass SVM?

Soft margin SVM?



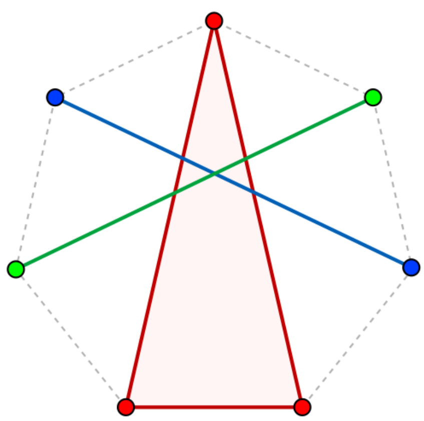


Image: Wikipedia, David Eppstein