Open Problems in Need of Computer Simulation

Henry Adams, Colorado State University

	$n=1$	2	3	4	5	6	7	8	9
$k=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$	$\vee^{7423} S^2$
3	$*$	$*$	$\vee^3 S^4$	$S^3 \vee (\vee^{24} S^4)$	$(\vee^9 S^3) \vee (\vee^{120} S^4)$				
4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}		2014,		Polymake	
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

Abstract: Though math papers are sometimes presented as "definition, lemma, theorem, proof", the ideas instead often arise via a computational experiment, which leads to a conjecture, which only later perhaps becomes a theorem. Such computations should be explained, advertised, acknowledged, and celebrated. I will share open problems where more computational experiments are needed, and where there is plenty of room for folks with algorithmic expertise to get involved. These open problems are related (for example) to nerve complexes, to Vietoris-Rips complexes, and to the persistent homology of fractals.

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1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
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4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}					
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

2014, Polymake

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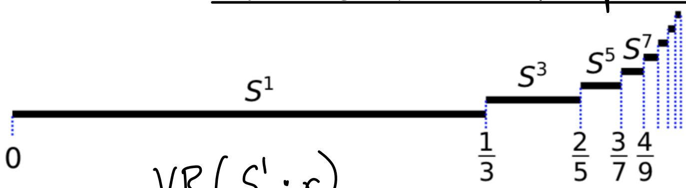
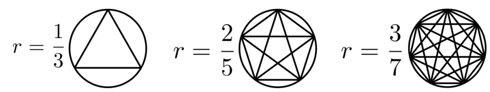
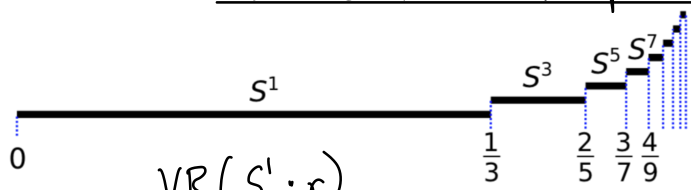
Vietoris-Rips of circle

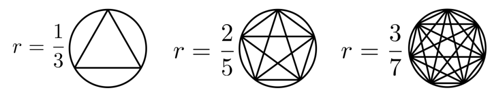


$VR(S^1; r)$



Vietoris – Rips of circle





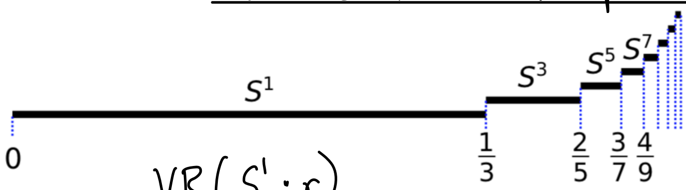
$VR(S^1; r)$

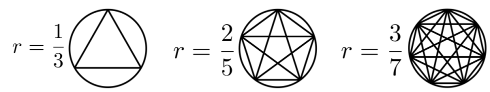
	k=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
n=4	1,1															
5	1,1															
6	1,1	2,1														
7	1,1	1,1														
8	1,1	1,1	3,1													
9	1,1	1,1	2,2													
10	1,1	1,1	1,1	4,1												
11	1,1	1,1	1,1	3,1												
12	1,1	1,1	1,1	2,3	5,1											
13	1,1	1,1	1,1	1,1	3,1											
14	1,1	1,1	1,1	1,1	3,1	6,1										
15	1,1	1,1	1,1	1,1	2,4	4,2										
16	1,1	1,1	1,1	1,1	1,1	3,1	7,1									
17	1,1	1,1	1,1	1,1	1,1	3,1	5,1									
18	1,1	1,1	1,1	1,1	1,1	2,5	3,1	8,1								
19	1,1	1,1	1,1	1,1	1,1	1,1	3,1	5,1								
20	1,1	1,1	1,1	1,1	1,1	1,1	3,1	4,3	9,1							
21	1,1	1,1	1,1	1,1	1,1	1,1	2,6	3,1	6,2							
22	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	5,1	10,1						
23	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	7,1						
24	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,7	3,1	5,1	11,1					
25	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	4,4	7,1					
26	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	12,1				
27	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,8	3,1	5,1	8,2				
28	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	6,3	13,1			
29	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	9,1			
30	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,9	3,1	4,5	7,1	14,1		
31	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	9,1		
32	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	7,1	15,1	
33	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,10	3,1	3,1		10,2	
34	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1			16,1
35	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	4,6	6,4	
36	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,11	3,1	3,1		8,3
37	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1		

Entry d, r in (row, col) = (n, k) means $VR(n \text{ even points}; k/n)$ has H_d of rank r.

2013, Javaplex

Vietoris–Rips of circle





$VR(S^1; r)$

	k=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
n=4	1,1															
5	1,1															
6	1,1	2,1														
7	1,1	1,1														
8	1,1	1,1	3,1													
9	1,1	1,1	2,2													
10	1,1	1,1	1,1	4,1												
11	1,1	1,1	1,1	3,1												
12	1,1	1,1	1,1	2,3	5,1											
13	1,1	1,1	1,1	1,1	3,1											
14	1,1	1,1	1,1	1,1	3,1	6,1										
15	1,1	1,1	1,1	1,1	2,4	4,2										
16	1,1	1,1	1,1	1,1	1,1	3,1	7,1									
17	1,1	1,1	1,1	1,1	1,1	3,1	5,1									
18	1,1	1,1	1,1	1,1	1,1	2,5	3,1	8,1								
19	1,1	1,1	1,1	1,1	1,1	1,1	3,1	5,1								
20	1,1	1,1	1,1	1,1	1,1	1,1	3,1	4,3	9,1							
21	1,1	1,1	1,1	1,1	1,1	1,1	2,6	3,1	6,2							
22	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	5,1	10,1						
23	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	7,1						
24	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,7	3,1	5,1	11,1					
25	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	4,4	7,1					
26	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	12,1				
27	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,8	3,1	5,1	8,2				
28	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	6,3	13,1			
29	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	9,1			
30	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,9	3,1	4,5	7,1	14,1		
31	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	9,1		
32	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	5,1	7,1	15,1	
33	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,10	3,1	3,1	[illegible]	10,2	
34	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	[illegible]	[illegible]	16,1
35	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	4,6	6,4	[illegible]
36	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	2,11	3,1	3,1	[illegible]	8,3
37	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	3,1	3,1	[illegible]	[illegible]

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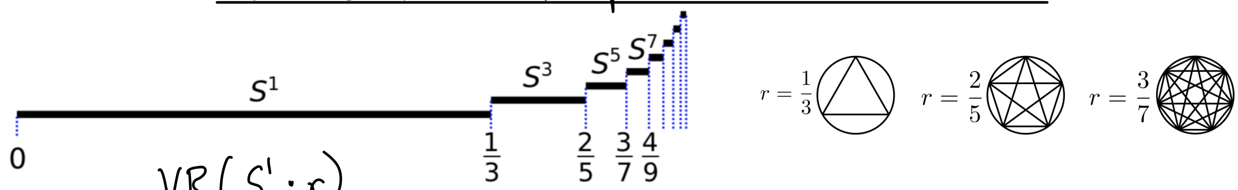


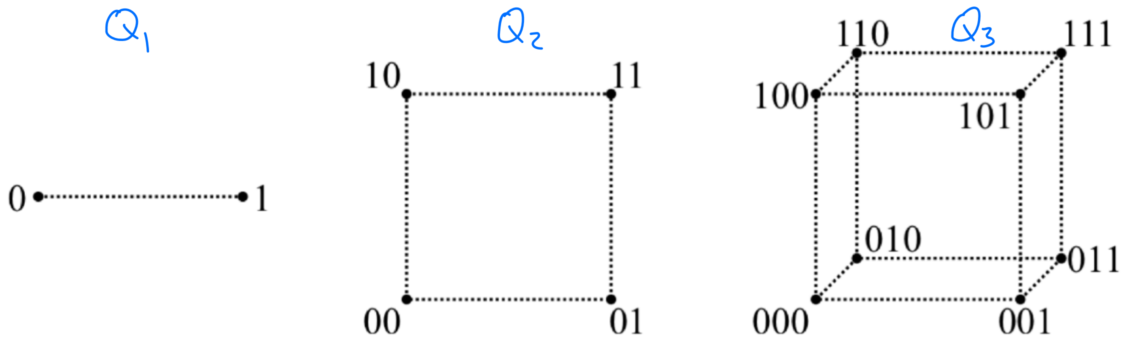
$VR(S^1; r)$

	$k=1$	2	3	4	5	6	7	8	9	10	11	12
$n=6$	S^1	S^2	$*$	$*$	$*$	$*$						
7	S^1	S^1	$*$	$*$	$*$	$*$						
8	S^1	S^1	S^3	$*$	$*$	$*$						
9	S^1	S^1	$\vee_2 S^2$	$*$	$*$	$*$						
10	S^1	S^1	S^1	S^4	$*$	$*$						
11	S^1	S^1	S^1	S^3	$*$	$*$						
12	S^1	S^1	S^1	$\vee_3 S^2$	S^5	$*$						
13	S^1	S^1	S^1	S^1	S^3	$*$						
14	S^1	S^1	S^1	S^1	S^3	S^6						
15	S^1	S^1	S^1	S^1	$\vee_4 S^2$	$\vee_2 S^4$						
16	S^1	S^1	S^1	S^1	S^1	S^3	S^7	$*$	$*$	$*$	$*$	$*$
17	S^1	S^1	S^1	S^1	S^1	S^3	S^5	$*$	$*$	$*$	$*$	$*$
18	S^1	S^1	S^1	S^1	S^1	$\vee_5 S^2$	S^3	S^8	$*$	$*$	$*$	$*$
19	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	$*$	$*$	$*$	$*$
20	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$\vee_3 S^4$	S^9	$*$	$*$	$*$
21	S^1	S^1	S^1	S^1	S^1	S^1	$\vee_6 S^2$	S^3	$\vee_2 S^6$	$*$	$*$	$*$
22	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^5	S^{10}	$*$	$*$
23	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^7	$*$	$*$
24	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$\vee_7 S^2$	S^3	S^5	S^{11}	$*$
25	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	$\vee_4 S^4$	S^7	$*$
26	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	S^5	S^{12}
27	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	$\vee_8 S^2$	S^3	S^5	$\vee_2 S^8$
28	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^1	S^3	S^3	$\vee_3 S^6$

Homotopy type of $VR(n \text{ even points}; k/n)$

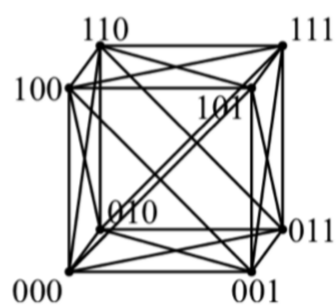
Vietoris-Rips of hypercube graphs

Def Let Q_n be the 2^n vertices of the hypercube graph, equipped with the shortest path metric.



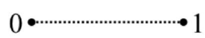
Def The Vietoris-Rips simplicial complex $VR(X;r)$ of metric space X at scale $r \geq 0$ has

- vertex set X
- finite simplex $\sigma \subseteq X$ when $\text{diameter}(\sigma) \leq r$.

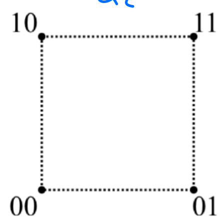


Vietoris-Rips of hypercube graphs

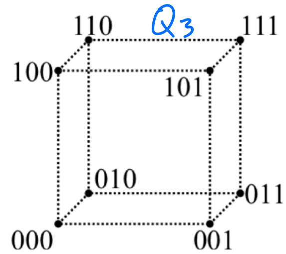
Q_1

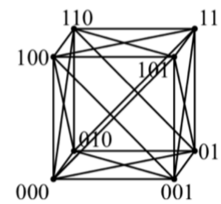


Q_2



Q_3

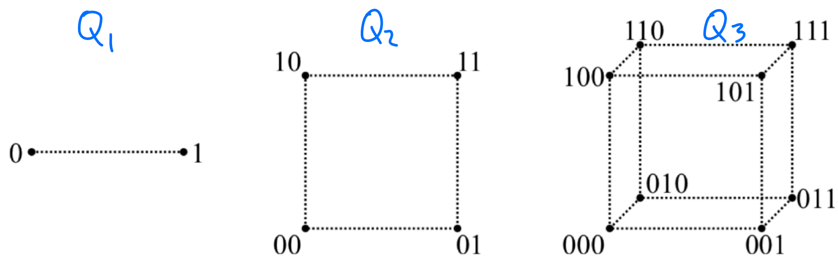


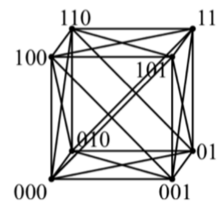


Homotopy types of $VR(Q_n; r)$

	$n=1$	2	3	4	5	6	7	8	9
$r=0$	S^0	$\bigvee^3 S^0$	$\bigvee^7 S^0$	$\bigvee^{15} S^0$	$\bigvee^{31} S^0$	$\bigvee^{63} S^0$	$\bigvee^{127} S^0$	$\bigvee^{255} S^0$	$\bigvee^{511} S^0$
1	$*$	S^1	$\bigvee^5 S^1$	$\bigvee^{17} S^1$	$\bigvee^{49} S^1$	$\bigvee^{129} S^1$	$\bigvee^{321} S^1$	$\bigvee^{769} S^1$	$\bigvee^{1793} S^1$
2	$*$	$*$	S^3	$\bigvee^9 S^3$	$\bigvee^{49} S^3$	$\bigvee^{209} S^3$	$\bigvee^{769} S^3$	$\bigvee^{2561} S^3$	$\bigvee^{7937} S^3$
3	$*$	$*$	$*$	S^7					
4	$*$	$*$	$*$	$*$	S^{15}				
5	$*$	$*$	$*$	$*$	$*$	S^{31}			
6	$*$	$*$	$*$	$*$	$*$	$*$	S^{63}		
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{127}	
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{255}

Vietoris-Rips of hypercube graphs





Homotopy types of $VR(Q_n; r)$

	$n=1$	2	3	4	5	6	7	8	9
$r=0$	S^0	$\bigvee^3 S^0$	$\bigvee^7 S^0$	$\bigvee^{15} S^0$	$\bigvee^{31} S^0$	$\bigvee^{63} S^0$	$\bigvee^{127} S^0$	$\bigvee^{255} S^0$	$\bigvee^{511} S^0$
1	$*$	S^1	$\bigvee^5 S^1$	$\bigvee^{17} S^1$	$\bigvee^{49} S^1$	$\bigvee^{129} S^1$	$\bigvee^{321} S^1$	$\bigvee^{769} S^1$	$\bigvee^{1793} S^1$
2	$*$	$*$	S^3	$\bigvee^9 S^3$	$\bigvee^{49} S^3$	$\bigvee^{209} S^3$	$\bigvee^{769} S^3$	$\bigvee^{2561} S^3$	$\bigvee^{7937} S^3$
3	$*$	$*$	$*$	S^7	$S^4 \vee \bigvee^{10} S^7$	$\bigvee^{11} S^4 \vee \bigvee^{60} S^7$	$\bigvee^{71} S^4 \vee \bigvee^{280} S^7$	$\bigvee^{351} S^4 \vee \bigvee^{1120} S^7$	$\bigvee^{1471} S^4 \vee \bigvee^{4032} S^7$
4	$*$	$*$	$*$	$*$	S^{15}				
5	$*$	$*$	$*$	$*$	$*$	S^{31}			
6	$*$	$*$	$*$	$*$	$*$	$*$	S^{63}		
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{127}	
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{255}

$H_i(\mathrm{VR}(Q_5;3);\mathbb{Z}) \cong \mathbb{Z}$ for $i=4$, $\cong \mathbb{Z}^{10}$ for $i=7$,

$H_i(\mathrm{VR}(Q_6;3);\mathbb{Z}) \cong \mathbb{Z}^{11}$ for $i=4$, $\cong \mathbb{Z}^{60}$ for $i=7$,

$H_i(\mathrm{VR}(Q_7;3);\mathbb{Z}) \cong \mathbb{Z}^{71}$ for $i=4$, $\cong \mathbb{Z}^{280}$ for $i=7$,

Polymake, 2014

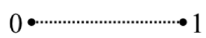
$H_i(\mathrm{VR}(Q_8;3);\frac{\mathbb{Z}}{2\mathbb{Z}}) \cong (\frac{\mathbb{Z}}{2\mathbb{Z}})^{351}$ for $i=4$, $\cong (\frac{\mathbb{Z}}{2\mathbb{Z}})^{1120}$ for $i=7$,

$H_i(\mathrm{VR}(Q_9;3);\frac{\mathbb{Z}}{2\mathbb{Z}}) \cong (\frac{\mathbb{Z}}{2\mathbb{Z}})^{1471}$ for $i=4$, $\cong (\frac{\mathbb{Z}}{2\mathbb{Z}})^{4032}$ for $i=7$,

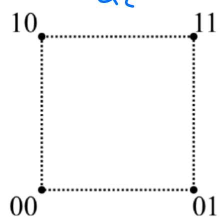
Ripser ++, 2020
Zhang, Xiao, Wang
Bauer

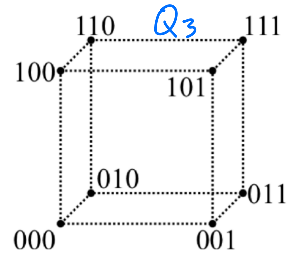
Vietoris-Rips of hypercube graphs

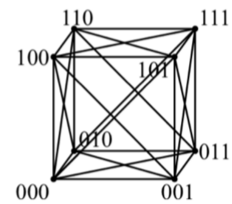
Q_1



Q_2







Homotopy types of $VR(Q_n; r)$

	$n=1$	2	3	4	5	6	7	8	9
$r=0$	S^0	$\bigvee^3 S^0$	$\bigvee^7 S^0$	$\bigvee^{15} S^0$	$\bigvee^{31} S^0$	$\bigvee^{63} S^0$	$\bigvee^{127} S^0$	$\bigvee^{255} S^0$	$\bigvee^{511} S^0$
1	$*$	S^1	$\bigvee^5 S^1$	$\bigvee^{17} S^1$	$\bigvee^{49} S^1$	$\bigvee^{129} S^1$	$\bigvee^{321} S^1$	$\bigvee^{769} S^1$	$\bigvee^{1793} S^1$
2	$*$	$*$	S^3	$\bigvee^9 S^3$	$\bigvee^{49} S^3$	$\bigvee^{209} S^3$	$\bigvee^{769} S^3$	$\bigvee^{2561} S^3$	$\bigvee^{7937} S^3$
3	$*$	$*$	$*$	S^7	$S^4 \vee \bigvee^{10} S^7$	$\bigvee^{11} S^4 \vee \bigvee^{60} S^7$	$\bigvee^{71} S^4 \vee \bigvee^{280} S^7$	$\bigvee^{351} S^4 \vee \bigvee^{1120} S^7$	$\bigvee^{1471} S^4 \vee \bigvee^{4032} S^7$
4	$*$	$*$	$*$	$*$	S^{15}				
5	$*$	$*$	$*$	$*$	$*$	S^{31}			
6	$*$	$*$	$*$	$*$	$*$	$*$	S^{63}		
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{127}	
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{255}

$H_i(\mathrm{VR}(Q_5;3);\mathbb{Z}) \cong \mathbb{Z}$ for $i=4$, $\cong \mathbb{Z}^{10}$ for $i=7$,

$H_i(\mathrm{VR}(Q_6;3);\mathbb{Z}) \cong \mathbb{Z}^{11}$ for $i=4$, $\cong \mathbb{Z}^{60}$ for $i=7$,

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Polymake, 2014

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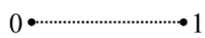
Ripser++, 2020
Zhang, Xiao, Wang
Bauer

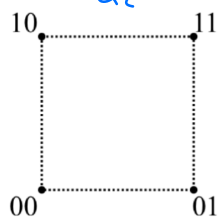
$2^{n-4}\binom{n}{4}$ is # of Q_4 subgraphs in Q_n.

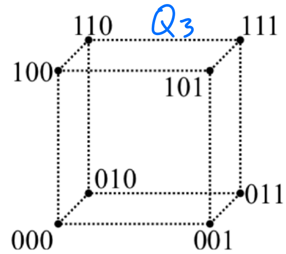
Vietoris-Rips of hypercube graphs

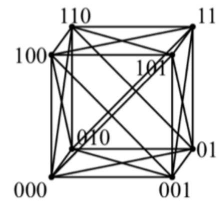
Q_1



Q_2







Homotopy types of $VR(Q_n; r)$

	$n=1$	2	3	4	5	6	7	8	9
$r=0$	S^0	$\bigvee^3 S^0$	$\bigvee^7 S^0$	$\bigvee^{15} S^0$	$\bigvee^{31} S^0$	$\bigvee^{63} S^0$	$\bigvee^{127} S^0$	$\bigvee^{255} S^0$	$\bigvee^{511} S^0$
1	$*$	S^1	$\bigvee^5 S^1$	$\bigvee^{17} S^1$	$\bigvee^{49} S^1$	$\bigvee^{129} S^1$	$\bigvee^{321} S^1$	$\bigvee^{769} S^1$	$\bigvee^{1793} S^1$
2	$*$	$*$	S^3	$\bigvee^9 S^3$	$\bigvee^{49} S^3$	$\bigvee^{209} S^3$	$\bigvee^{769} S^3$	$\bigvee^{2561} S^3$	$\bigvee^{7937} S^3$
3	$*$	$*$	$*$	S^7	$S^4 \vee \bigvee^{10} S^7$	$\bigvee^{11} S^4 \vee \bigvee^{60} S^7$	$\bigvee^{71} S^4 \vee \bigvee^{280} S^7$	$\bigvee^{351} S^4 \vee \bigvee^{1120} S^7$	$\bigvee^{1471} S^4 \vee \bigvee^{4032} S^7$
4	$*$	$*$	$*$	$*$	S^{15}				
5	$*$	$*$	$*$	$*$	$*$	S^{31}			
6	$*$	$*$	$*$	$*$	$*$	$*$	S^{63}		
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{127}	
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{255}

$H_i(\mathrm{VR}(Q_5;3);\mathbb{Z}) \cong \mathbb{Z}$ for $i=4$, $\cong \mathbb{Z}^{10}$ for $i=7$,

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Polymake, 2014

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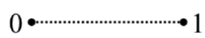
Ripser++, 2020 Zhang, Xiao, Wang Bauer

4-dim'l skeleton of n-cube polytope is $\left(\sum_{i=4}^{n-1} 2^{i-4}\binom{i}{4}\right)$-fold wedge sum of 4-spheres

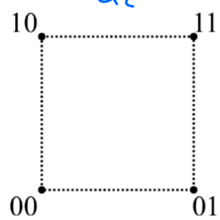
$2^{n-4}\binom{n}{4}$ is # of Q_4 subgraphs in Q_n.

Vietoris-Rips of hypercube graphs

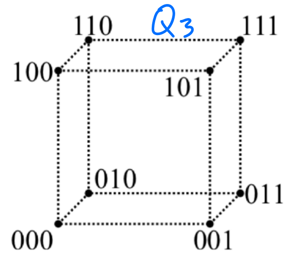
Q_1

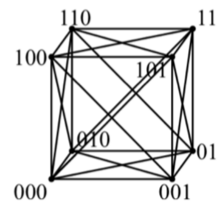


Q_2



Q_3





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1	$*$	S^1	$\bigvee^5 S^1$	$\bigvee^{17} S^1$	$\bigvee^{49} S^1$	$\bigvee^{129} S^1$	$\bigvee^{321} S^1$	$\bigvee^{769} S^1$	$\bigvee^{1793} S^1$
2	$*$	$*$	S^3	$\bigvee^9 S^3$	$\bigvee^{49} S^3$	$\bigvee^{209} S^3$	$\bigvee^{769} S^3$	$\bigvee^{2561} S^3$	$\bigvee^{7937} S^3$
3	$*$	$*$	$*$	S^7	$S^4 \vee \bigvee^{10} S^7$	$\bigvee^{11} S^4 \vee \bigvee^{60} S^7$	$\bigvee^{71} S^4 \vee \bigvee^{280} S^7$	$\bigvee^{351} S^4 \vee \bigvee^{1120} S^7$	$\bigvee^{1471} S^4 \vee \bigvee^{4032} S^7$
4	$*$	$*$	$*$	$*$	S^{15}				
5	$*$	$*$	$*$	$*$	$*$	S^{31}			
6	$*$	$*$	$*$	$*$	$*$	$*$	S^{63}		
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{127}	
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{255}

$$H_i(\mathrm{VR}(Q_5;3);\mathbb{Z}) \cong \mathbb{Z} \text{ for } i=4, \quad \cong \mathbb{Z}^{10} \text{ for } i=7,$$
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Polymake, 2014

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Ripser++, 2020 Zhang, Xiao, Wang Bauer

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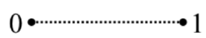
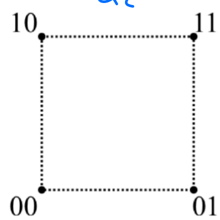
Vietoris-Rips of hypercube graphs

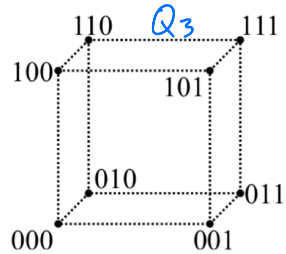
Q_1

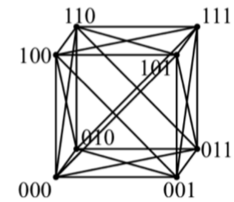


Q_2



Q_3





Homotopy types of $VR(Q_n; r)$

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1	$*$	S^1	$\bigvee^5 S^1$	$\bigvee^{17} S^1$	$\bigvee^{49} S^1$	$\bigvee^{129} S^1$	$\bigvee^{321} S^1$	$\bigvee^{769} S^1$	$\bigvee^{1793} S^1$
2	$*$	$*$	S^3	$\bigvee^9 S^3$	$\bigvee^{49} S^3$	$\bigvee^{209} S^3$	$\bigvee^{769} S^3$	$\bigvee^{2561} S^3$	$\bigvee^{7937} S^3$
3	$*$	$*$	$*$	S^7	$S^4 \vee \bigvee^{10} S^7$	$\bigvee^{11} S^4 \vee \bigvee^{60} S^7$	$\bigvee^{71} S^4 \vee \bigvee^{280} S^7$	$\bigvee^{351} S^4 \vee \bigvee^{1120} S^7$	$\bigvee^{1471} S^4 \vee \bigvee^{4032} S^7$
4	$*$	$*$	$*$	$*$	S^{15}				
5	$*$	$*$	$*$	$*$	$*$	S^{31}			
6	$*$	$*$	$*$	$*$	$*$	$*$	S^{63}		
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{127}	
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^{255}

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Polymake, 2014

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Ripser ++, 2020 Zhang, Xiao, Wang Bauer

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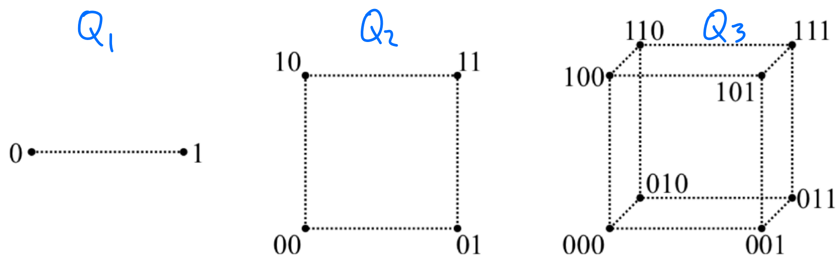
Conjecture: Is $\mathrm{rank}(H_{2^r-1}(VR(Q_n;r))) = 2^{n-r-1}\binom{n}{r+1}$ for $r \geq 3$?

Q_{r+1} in Q_n

Čech of hypercube graphs

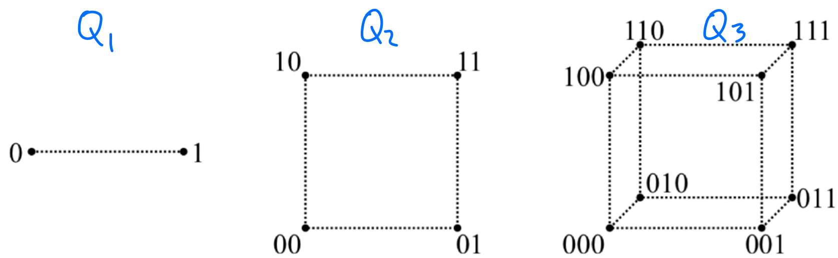


Homotopy types of Čech$(Q_n; k)$?

	$n=1$	2	3	4	5	6	7	8	9
$k=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$	$\vee^{7423} S^2$
3	$*$	$*$	$\vee^3 S^4$	$S^3 \vee (\vee^{24} S^4)$	$(\vee^9 S^3) \vee (\vee^{120} S^4)$				
4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}					
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

2014, Polymake

Čech of hypercube graphs

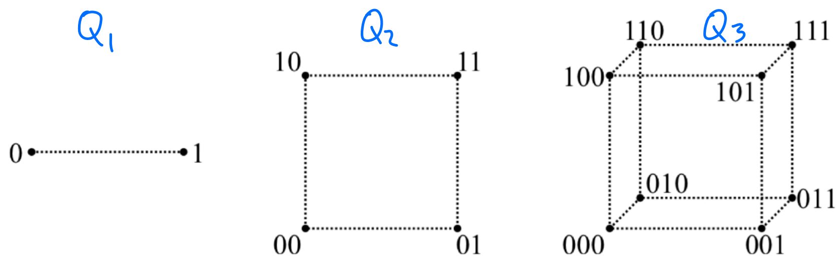


Homotopy types of $\check{C}ech(Q_n; k)$?

	$n=1$	2	3	4	5	6	7	8	9
$k=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$	$\vee^{7423} S^2$
3	$*$	$*$	$\vee^3 S^4$	$S^3 \vee (\vee^{24} S^4)$	$(\vee^9 S^3) \vee (\vee^{120} S^4)$				
4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}					
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

2014, Polymake

Čech of hypercube graphs



Homotopy types of Čech$(Q_n; k)$?

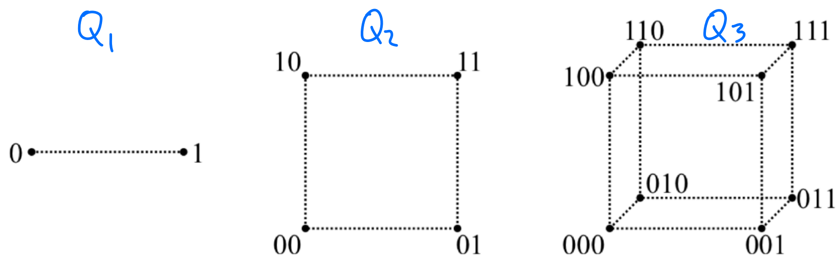
	$n=1$	2	3	4	5	6	7	8	9
$k=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$	$\vee^{7423} S^2$
3	$*$	$*$	$\vee^3 S^4$	$S^3 \vee (\vee^{24} S^4)$	$(\vee^9 S^3) \vee (\vee^{120} S^4)$				
4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}					
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

2014, Polymake

Homotopy type of the r-skeleton of the n-cube polytope, which for $r<n$ is a $\left(\sum_{i=r}^{n-1} 2^{i-r}\binom{i}{r}\right)$-fold wedge sum of S^r!

	$n=1$	2	3	4	5	6	7	8	9
$r=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$
3	$*$	$*$	$*$	S^3	$\vee^9 S^3$	$\vee^{49} S^3$	$\vee^{209} S^3$	$\vee^{769} S^3$	$\vee^{2561} S^3$
4	$*$	$*$	$*$	$*$	S^4	$\vee^{11} S^4$	$\vee^{71} S^4$	$\vee^{351} S^4$	$\vee^{1471} S^4$
5	$*$	$*$	$*$	$*$	$*$	S^5	$\vee^{13} S^5$	$\vee^{97} S^5$	$\vee^{545} S^5$
6	$*$	$*$	$*$	$*$	$*$	$*$	S^6	$\vee^{15} S^6$	$\vee^{127} S^6$
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^7	$\vee^{17} S^7$
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^8

Čech of hypercube graphs



Homotopy types of Čech$(Q_n;k)$?

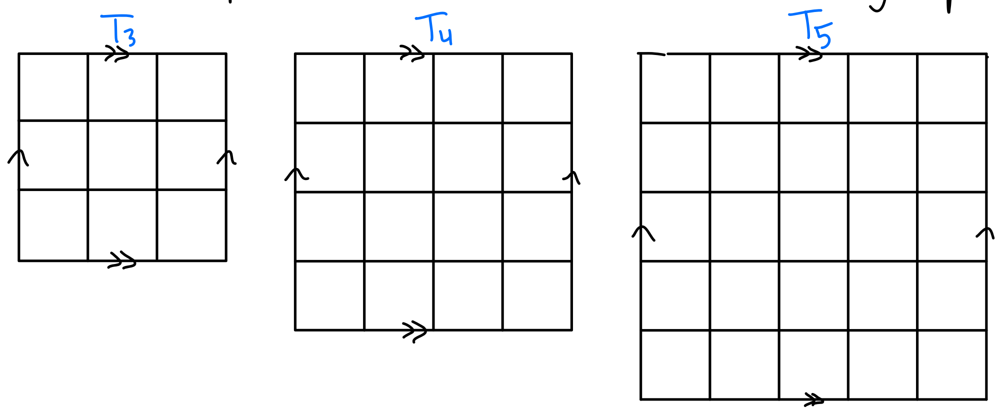
	$n=1$	2	3	4	5	6	7	8	9
$k=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$	$\vee^{7423} S^2$
3	$*$	$*$	$\vee^3 S^4$	$S^3 \vee (\vee^{24} S^4)$	$(\vee^9 S^3) \vee (\vee^{120} S^4)$				
4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}					
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

2014, Polymake

Homotopy type of the r-skeleton of the n-cube polytope, which for $r<n$ is a $\left(\sum_{i=r}^{n-1} 2^{i-r}\binom{i}{r}\right)$-fold wedge sum of S^r!

	$n=1$	2	3	4	5	6	7	8	9
$r=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$
3	$*$	$*$	$*$	S^3	$\vee^9 S^3$	$\vee^{49} S^3$	$\vee^{209} S^3$	$\vee^{769} S^3$	$\vee^{2561} S^3$
4	$*$	$*$	$*$	$*$	S^4	$\vee^{11} S^4$	$\vee^{71} S^4$	$\vee^{351} S^4$	$\vee^{1471} S^4$
5	$*$	$*$	$*$	$*$	$*$	S^5	$\vee^{13} S^5$	$\vee^{97} S^5$	$\vee^{545} S^5$
6	$*$	$*$	$*$	$*$	$*$	$*$	S^6	$\vee^{15} S^6$	$\vee^{127} S^6$
7	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^7	$\vee^{17} S^7$
8	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	S^8

Vietoris-Rips and Čech of torus graphs



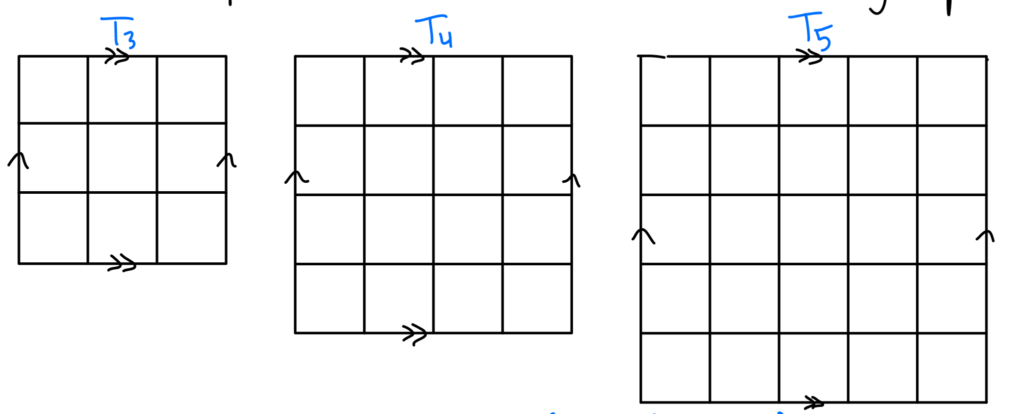
Homology of $VR(T_n; k)$

	$k=0$	1	2	3	4	5
$n=3$	$\vee^{8}S^0$	$\vee^{4}S^1$	$*$	$*$	$*$	$*$
4	$\vee^{15}S^0$	$\vee^{17}S^1$	$H_3=9$	$H_7=1$	$*$	$*$
5	$\vee^{24}S^0$	$\vee^{26}S^1$	$H_2=9$	$H_4=9$	$*$	$*$
6	$\vee^{35}S^0$	$\vee^{37}S^1$	$H_2=23$	$H_3=1, H_5=12$	$H_5=23, H_8=2$	$H_{17}=1(crosspltp.)$
7	$\vee^{48}S^0$	$\vee^{50}S^1$	$\mathbb{T}^2$	$H_3=1, H_4=14$	$H_3=1$	$H_2=0, (H_3=??)$
8	$\vee^{63}S^0$	$\vee^{65}S^1$	$\mathbb{T}^2$	$H_2=15, H_3=16$	$H_3=1, H_7=16$	$H_2=0, (H_{\geq 3}=??)$
9	$\vee^{80}S^0$	$\vee^{82}S^1$	$\mathbb{T}^2$	$H_2=53$	$H_3=1, H_5=36$	$H_2=0, H_3=1, (H_{\geq 4}=??)$
10	$\vee^{99}S^0$	$\vee^{101}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$H_3=21, H_4=60$	$H_3=1, H_9=20$
11	$\vee^{120}S^0$	$\vee^{122}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$H_2=21, H_3=22$	$H_3=1, H_6=22$
12	$\vee^{143}S^0$	$\vee^{145}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$H_2=95$	$H_2=0, H_3=1, (H_{\geq 4}=??)$
13	$\vee^{160}S^0$	$\vee^{170}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	$H_2=0, H_3=(Z/2)^{24}+Z^3, (H_{\geq 4}=??)$
14	$\vee^{195}S^0$	$\vee^{197}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	$H_2=27, (H_{\geq 3}=??)$
15	$\vee^{224}S^0$	$\vee^{226}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	$H_2=149, (H_{\geq 3}=??)$
16	$\vee^{255}S^0$	$\vee^{257}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	
17	$\vee^{288}S^0$	$\vee^{290}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	
18	$\vee^{323}S^0$	$\vee^{325}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	Polymake, 2014
19	$\vee^{360}S^0$	$\vee^{362}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	
20	$\vee^{399}S^0$	$\vee^{401}S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$	$\mathbb{T}^2$	

$\mathbb{T}^2$ means homology of torus

$H_i = d$ is shorthand for $H_i \cong \mathbb{Z}^d$

Vietoris-Rips and Čech of torus graphs



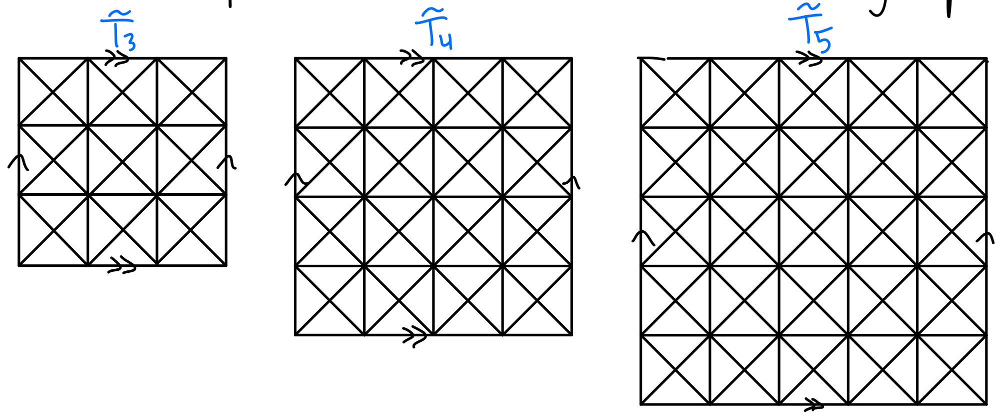
Homology of Čech$(T_n; k)$

	$k=0$	1	2	3	4	5	6	7
$n=3$	$\vee^{8}S^0$	$\vee^{10}S^1$	$H_2=14$	$H_5=4$	$*$	$*$	$*$	$*$
4	$\vee^{15}S^0$	$\vee^{17}S^1$	$H_2=31$	$H_3=1, H_4=24$	$H_4=1, H_6=10$	$H_{10}=7$	$H_{14}=1$	$*$
5	$\vee^{24}S^0$	$\vee^{26}S^1$	$H_1=2, H_3=26$	$H_2=9, H_3=10$	$H_3=1, H_4=10$	$H_5=16$	$H_8=9$	
6	$\vee^{35}S^0$	$\vee^{37}S^1$	$H_1=2, H_2=37$	$H_2=35$	$H_3=13, H_4=36$	$H_3=1, H_7=12$		
7	$\vee^{48}S^0$	$\vee^{50}S^1$	$H_1=2, H_2=50$	$\mathbb{T}^2$	$H_2=13, H_3=14$			
8	$\vee^{63}S^0$	$\vee^{65}S^1$	$H_1=2, H_2=65$	$\mathbb{T}^2$	$H_2=63$			
9	$\vee^{80}S^0$	$\vee^{82}S^1$	$H_1=2, H_2=82$	$\mathbb{T}^2$	$\mathbb{T}^2$			
10	$\vee^{99}S^0$	$\vee^{101}S^1$	$H_1=2, H_2=101$	$\mathbb{T}^2$	$\mathbb{T}^2$			
11	$\vee^{120}S^0$	$\vee^{122}S^1$	$H_1=2, H_2=122$	$\mathbb{T}^2$	$\mathbb{T}^2$			
12	$\vee^{143}S^0$	$\vee^{145}S^1$	$H_1=2, H_2=145$	$\mathbb{T}^2$	$\mathbb{T}^2$			
13	$\vee^{160}S^0$	$\vee^{170}S^1$	$H_1=2, H_2=170$	$\mathbb{T}^2$	$\mathbb{T}^2$			
14	$\vee^{195}S^0$	$\vee^{197}S^1$	$H_1=2, H_2=197$	$\mathbb{T}^2$	$\mathbb{T}^2$			
15	$\vee^{224}S^0$	$\vee^{226}S^1$	$H_1=2, H_2=226$	$\mathbb{T}^2$	$\mathbb{T}^2$			
16	$\vee^{255}S^0$	$\vee^{257}S^1$	$H_1=2, H_2=257$	$\mathbb{T}^2$	$\mathbb{T}^2$	Polymake, 2014		
17	$\vee^{288}S^0$	$\vee^{290}S^1$	$H_1=2, H_2=290$	$\mathbb{T}^2$	$\mathbb{T}^2$			
18	$\vee^{323}S^0$	$\vee^{325}S^1$	$H_1=2, H_2=325$	$\mathbb{T}^2$	$\mathbb{T}^2$			
19	$\vee^{360}S^0$	$\vee^{362}S^1$	$H_1=2, H_2=362$	$\mathbb{T}^2$	$\mathbb{T}^2$			
20	$\vee^{399}S^0$	$\vee^{401}S^1$	$H_1=2, H_2=401$	$\mathbb{T}^2$	$\mathbb{T}^2$			

$\mathbb{T}^2$ means homology of torus

$H_i = d$ is shorthand for $H_i \cong \mathbb{Z}^d$

Vietoris-Rips and Čech of torus graphs



Homology of $VR(\widetilde{T}_n; k)$

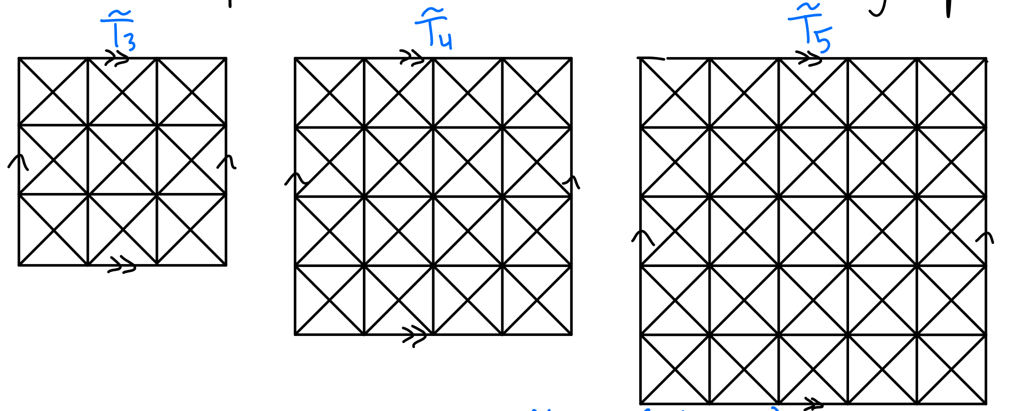
	$k=0$	1	2	3	4	5	6	7	8
$n=3$	$\vee^8 S^0$	$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
4	$\vee^{15} S^0$	$\mathbb{T}^2$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
5	$\vee^{24} S^0$	$\mathbb{T}^2$							
6	$\vee^{35} S^0$	$\mathbb{T}^2$							
7	$\vee^{48} S^0$	$\mathbb{T}^2$							
8	$\vee^{63} S^0$	$\mathbb{T}^2$							
9	$\vee^{80} S^0$	$\mathbb{T}^2$							
10	$\vee^{99} S^0$	$\mathbb{T}^2$							
11	$\vee^{120} S^0$	$\mathbb{T}^2$							
12	$\vee^{143} S^0$	$\mathbb{T}^2$							

Polymake, 2014

$\mathbb{T}^2$ means homology of torus

$H_i = d$ is shorthand for $H_i \cong \mathbb{Z}^d$

Vietoris-Rips and Čech of torus graphs



Homology of $\check{C}ech(\tilde{T}_n; k)$

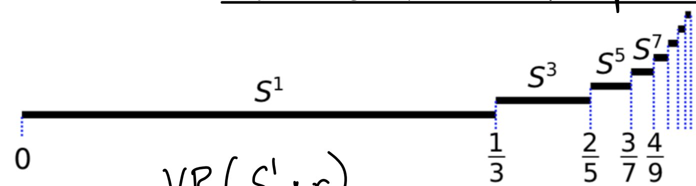
	$k=0$	1	2	3	4	5	6	7	8
$n=3$	$\vee^{8} S^0$	$\vee^{28} S^1$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
4	$\vee^{15} S^0$	$\vee^{49} S^1$	$H_2 = 2, H_4 = 1$	$H_8 = 4, H_{12} = 1$	$*$				
5	$\vee^{24} S^0$	$\vee^{76} S^1$	$\mathbb{T}^2$	$H_3 = 2, H_4 = \mathbb{Z}/2 \oplus \mathbb{Z}^{26}, H_5 = 75$					
6	$\vee^{35} S^0$	$\vee^{109} S^1$	$\mathbb{T}^2$	$H_2 = 4, H_3 = 23$					
7	$\vee^{48} S^0$	$\vee^{148} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
8	$\vee^{63} S^0$	$\vee^{193} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
9	$\vee^{80} S^0$	$\vee^{244} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
10	$\vee^{99} S^0$	$\vee^{301} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
11	$\vee^{120} S^0$	$\vee^{364} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
12	$\vee^{143} S^0$	$\vee^{433} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
13	$\vee^{160} S^0$	$\vee^{508} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
14	$\vee^{195} S^0$	$\vee^{589} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
15	$\vee^{224} S^0$	$\vee^{676} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
16	$\vee^{255} S^0$	$\vee^{769} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
17	$\vee^{288} S^0$	$\vee^{868} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
18	$\vee^{323} S^0$	$\vee^{973} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
19	$\vee^{360} S^0$	$\vee^{1084} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					
20	$\vee^{399} S^0$	$\vee^{1201} S^1$	$\mathbb{T}^2$	$\mathbb{T}^2$					

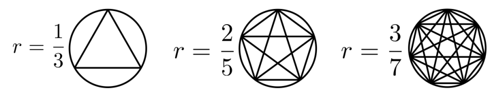
Polymake, 2014

$\mathbb{T}^2$ means homology of torus

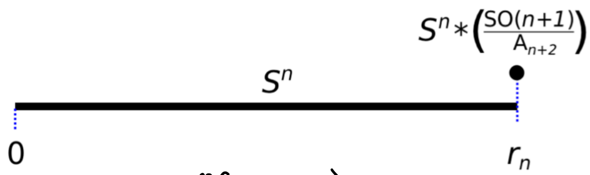
$H_i = d$ is shorthand for $H_i \cong \mathbb{Z}^d$

Vietoris-Rips of spheres





$\mathrm{VR}(S^1; r)$



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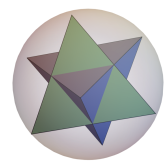
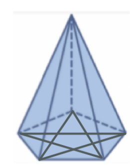
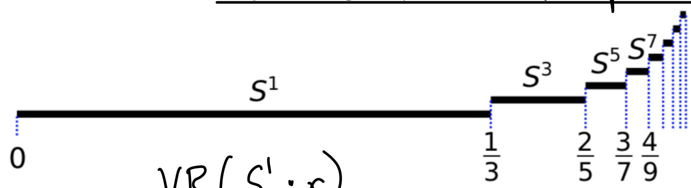
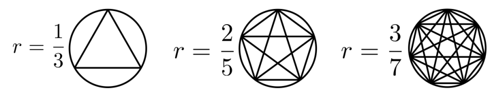




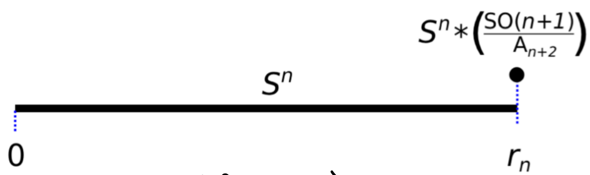
$\mathrm{VR}^m(S^n; r)$

Vietoris–Rips of spheres





$\mathrm{VR}(S^1; r)$



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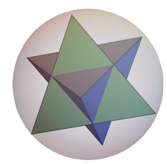
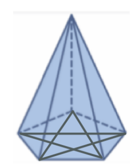




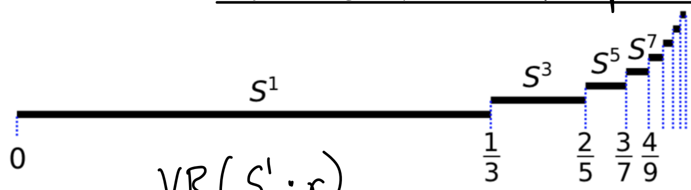
$\mathrm{VR}^m(S^n; r)$

We have $\mathrm{VR}^m(S^2; r_2) \simeq S^2 * \frac{SO(3)}{A_4}$ with

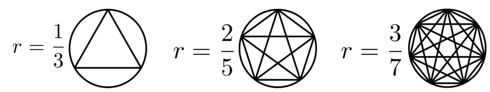
$$\tilde{H}_i\left(S^2 * \frac{SO(3)}{A_4}\right) = \tilde{H}_{i-3}\left(\frac{SO(3)}{A_4}\right) = \begin{cases} \mathbb{Z}/3\mathbb{Z} & \text{if } i = 4 \\ \mathbb{Z} & \text{if } i = 6 \\ 0 & \text{otherwise.} \end{cases}$$

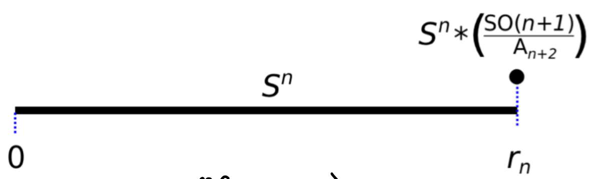
For a dense enough $X \subset S^2$, can you compute $H_4(\mathrm{VR}(X; r_2 + \varepsilon)) \cong \mathbb{Z}/3\mathbb{Z}$?

Vietoris–Rips of spheres



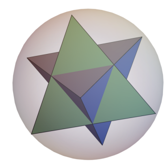
$VR(S^1;r)$

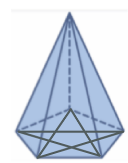




$VR^m(S^n;r)$

???





We have $VR^m(S^2;r_2) \simeq S^2 * \frac{SO(3)}{A_4}$ with

$$\tilde{H}_i\left(S^2 * \tfrac{SO(3)}{A_4}\right) = \tilde{H}_{i-3}\left(\tfrac{SO(3)}{A_4}\right) = \begin{cases} \mathbb{Z}/3\mathbb{Z} & \text{if } i=4 \\ \mathbb{Z} & \text{if } i=6 \\ 0 & \text{otherwise.} \end{cases}$$

For a dense enough $X \subset S^2$, can you compute $H_4(VR(X;r_2+\varepsilon)) \cong \mathbb{Z}/3\mathbb{Z}$?

Homotopy Connectivity $(VR(S^n;r))$

	$r=0$		$r=\pi$
$n=0$	-1		∞
$n=1$	0	$\frac{2\pi}{3}$ 2 $\frac{4\pi}{5}$ 4 $\frac{6\pi}{7}$ …	∞
$n=2$	1	r_2 3	∞
$n=3$	2	r_3 4	∞
$n=4$	3	r_4 5	∞

Open Problems in Need of Computer Simulation

Henry Adams, Colorado State University

	$n=1$	2	3	4	5	6	7	8	9
$k=0$	S^0	$\vee^3 S^0$	$\vee^7 S^0$	$\vee^{15} S^0$	$\vee^{31} S^0$	$\vee^{63} S^0$	$\vee^{127} S^0$	$\vee^{255} S^0$	$\vee^{511} S^0$
1	$*$	S^1	$\vee^5 S^1$	$\vee^{17} S^1$	$\vee^{49} S^1$	$\vee^{129} S^1$	$\vee^{321} S^1$	$\vee^{769} S^1$	$\vee^{1793} S^1$
2	$*$	S^2	$\vee^7 S^2$	$\vee^{31} S^2$	$\vee^{111} S^2$	$\vee^{351} S^2$	$\vee^{1023} S^2$	$\vee^{2815} S^2$	$\vee^{7423} S^2$
3	$*$	$*$	$\vee^3 S^4$	$S^3 \vee (\vee^{24} S^4)$	$(\vee^9 S^3) \vee (\vee^{120} S^4)$				
4	$*$	$*$	S^6	$S^4 \vee (\vee^{10} S^6)$	$(\vee^{11} S^4) \vee (\vee^{60} S^6)$				
5	$*$	$*$	$*$	$\vee^7 S^{10}$					
6	$*$	$*$	$*$	S^{14}					
7	$*$	$*$	$*$	$*$					
8	$*$	$*$	$*$	$*$	S^{30}				

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Abstract: Though math papers are sometimes presented as "definition, lemma, theorem, proof", the ideas instead often arise via a computational experiment, which leads to a conjecture, which only later perhaps becomes a theorem. Such computations should be explained, advertised, acknowledged, and celebrated. I will share open problems where more computational experiments are needed, and where there is plenty of room for folks with algorithmic expertise to get involved. These open problems are related (for example) to nerve complexes, to Vietoris-Rips complexes, and to the persistent homology of fractals.

Thanks!