

From persistent homology  
to machine learning



Henry Adams

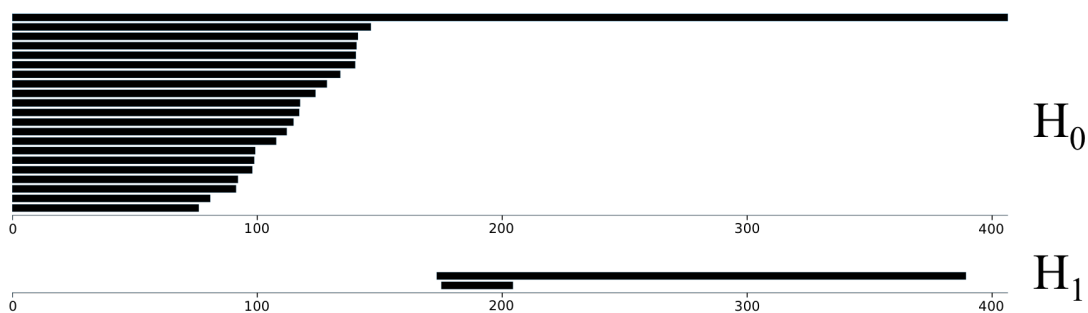
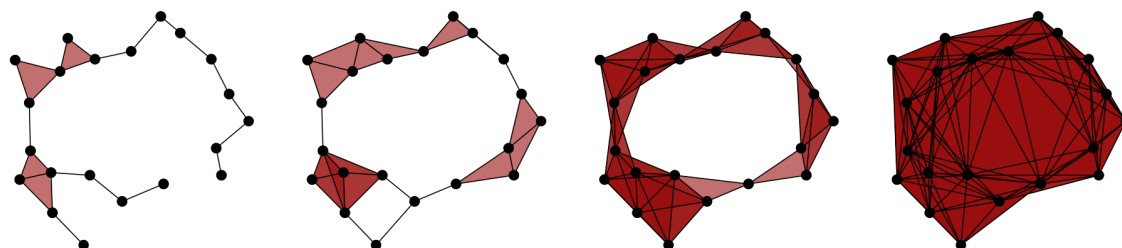
Colorado State University

# From persistent homology to machine learning

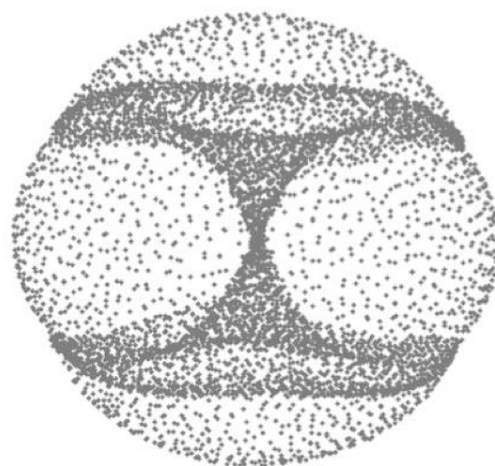
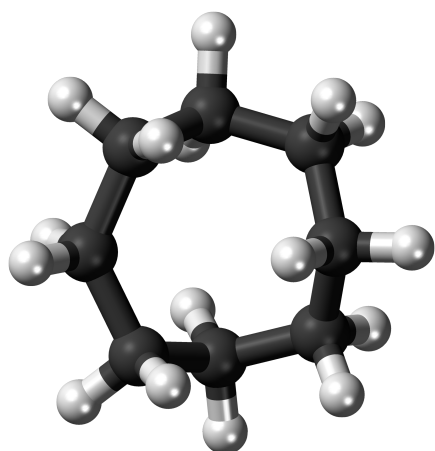


Persistent homology measures both the global topology and the local geometry of a dataset.

# Global topology



# Global topology

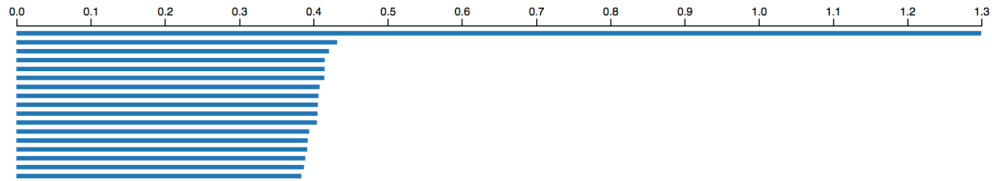


Topology of cyclo-octane energy landscape  
Martin, Thompson, Coutsiaris, Watson, 2010

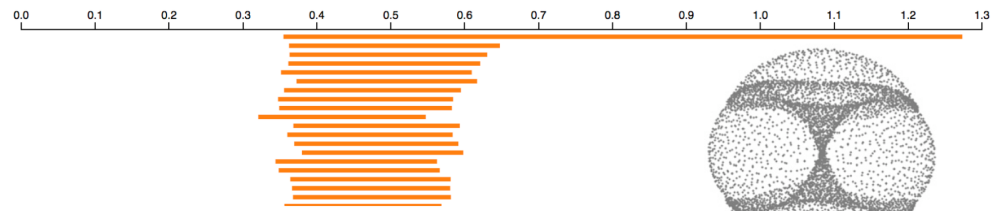


# Global topology

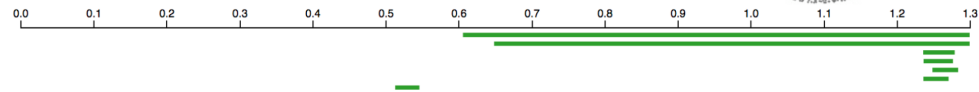
Persistence intervals in dimension 0:



Persistence intervals in dimension 1:

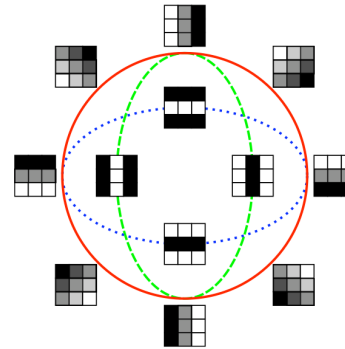
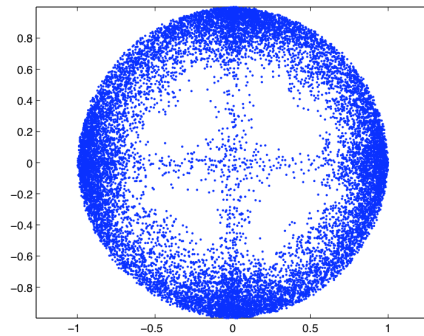


Persistence intervals in dimension 2:



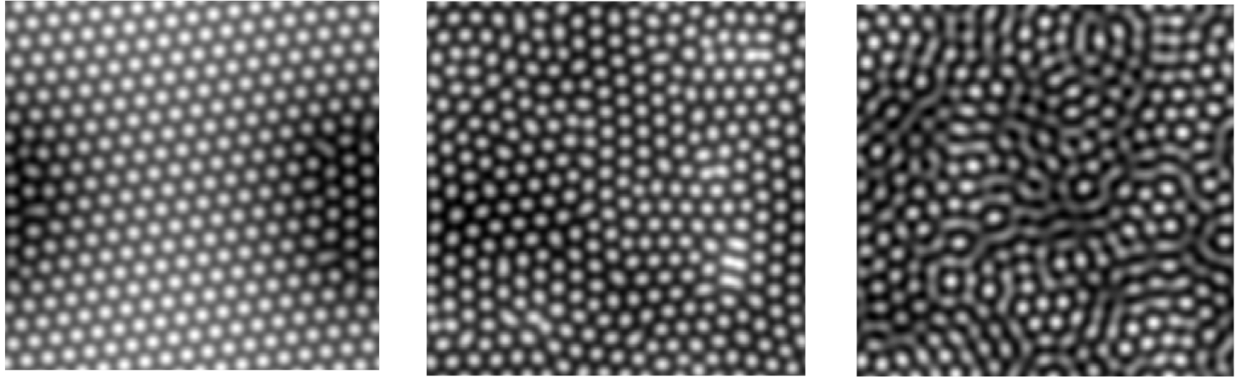
Topology of cyclo-octane energy landscape  
Martin, Thompson, Coutsiaris, Watson, 2010

# Global topology



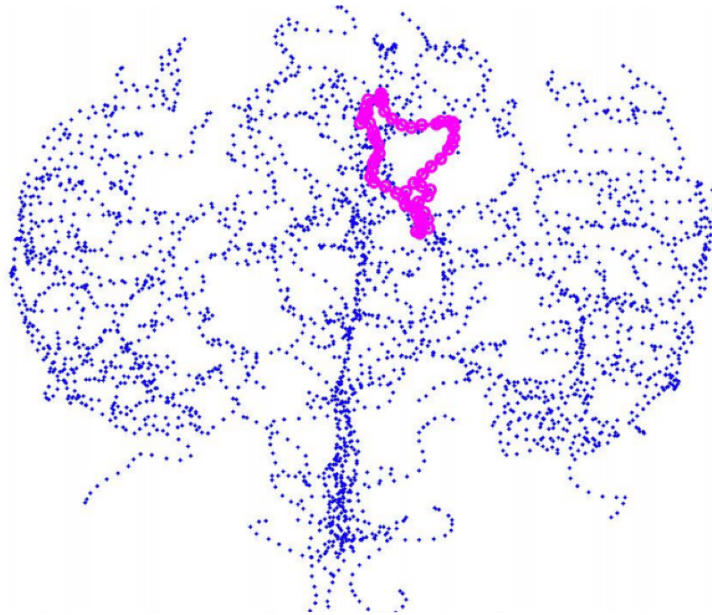
On the local behavior of natural images  
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008

# Local geometry



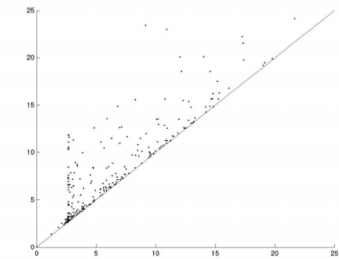
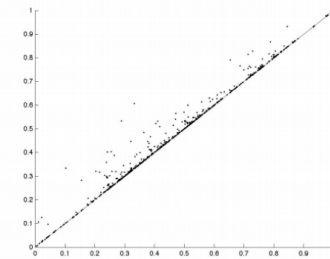
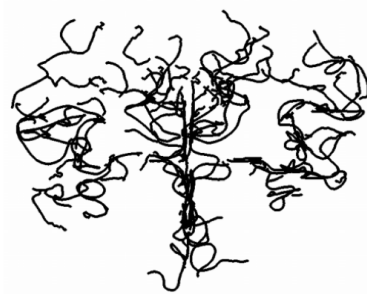
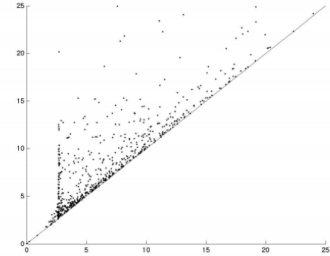
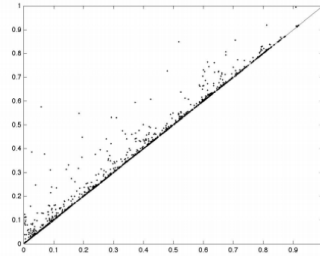
Measures of order for nearly hexagonal lattices  
Motta, Neville, Shipman, Pearson, Bradley, 2018

# Local geometry



Persistent homology analysis of brain artery trees  
Bendich, Marron, Miller, Pieloch, Skwerer, 2014

# Local geometry



Persistent homology analysis of brain artery trees  
Bendich, Marron, Miller, Pieloch, Skwerer, 2014

Local geometry



Collective motion, self-organization

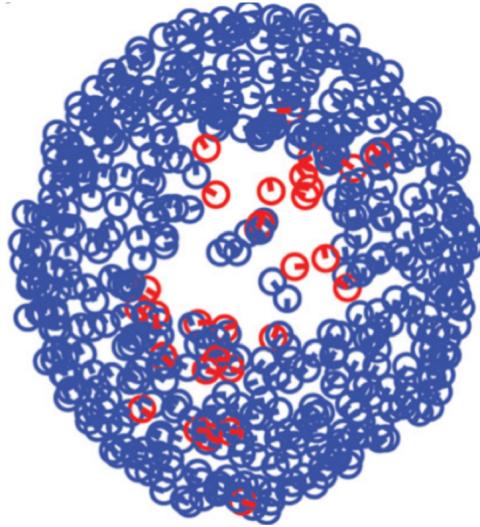
# Local geometry



Collective motion, self-organization

Topological data analysis of biological aggregation models  
Topaz, Ziegelmeier, Halverson, 2015

Local geometry

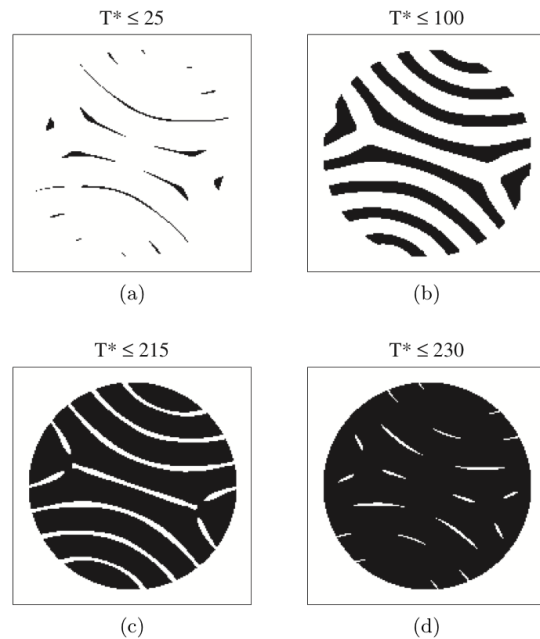


Collective motion, self-organization

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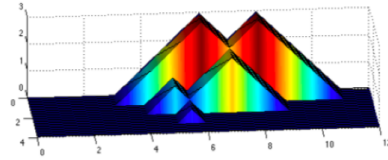
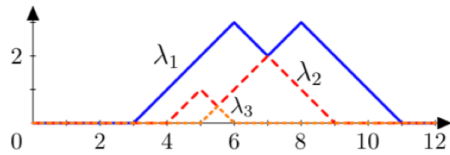
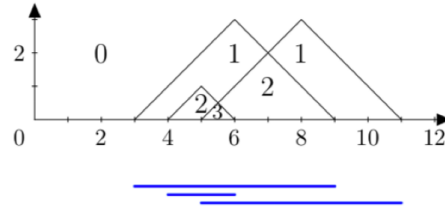
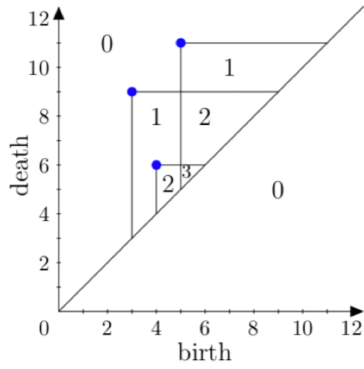


# Local geometry



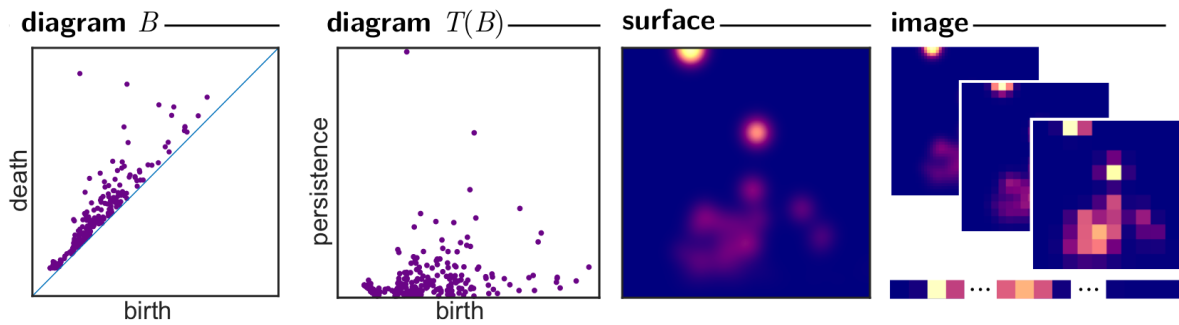
Analysis of Kolmogorov flow and Rayleigh-Bénard convection  
using persistent homology  
Kramár, Levanger, Tithof, Suri, Xu, Paul, Schatz, Mischaikow

# Local geometry



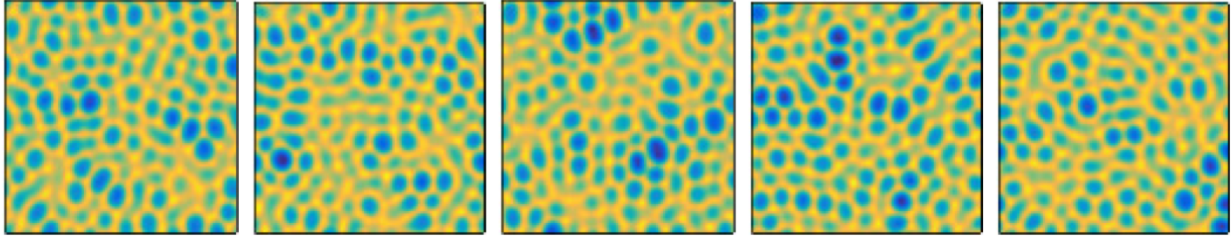
Statistical topological data analysis using persistence landscapes  
Bubenik, 2015

# Local geometry



Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017

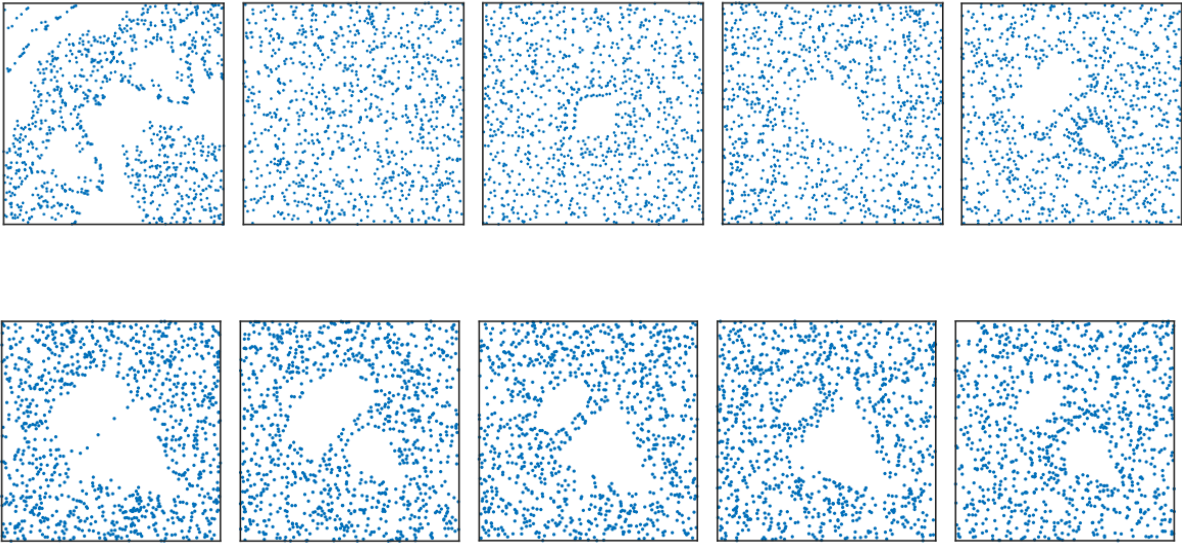
# Local geometry



ANSWER: (from left)  $r = 1.75, 2, 1.75, 2, 2$ .

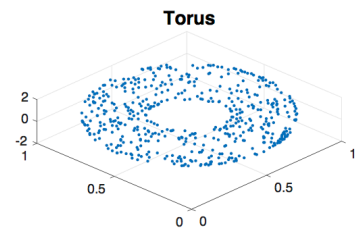
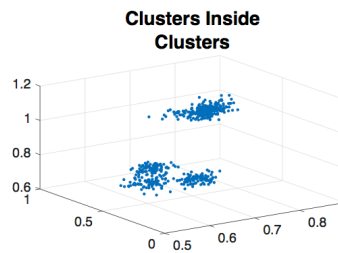
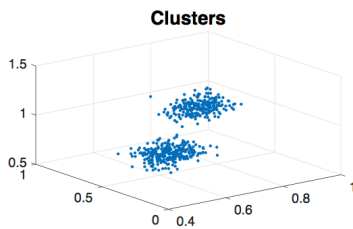
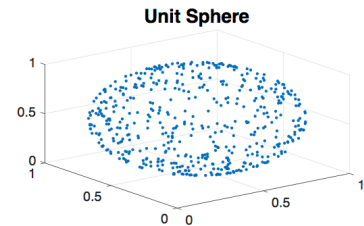
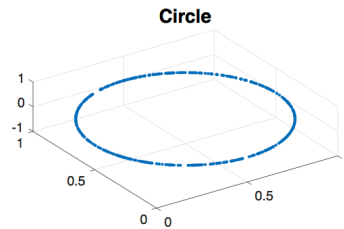
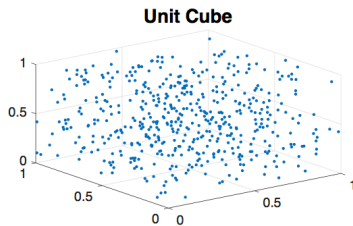
Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017

## Local geometry



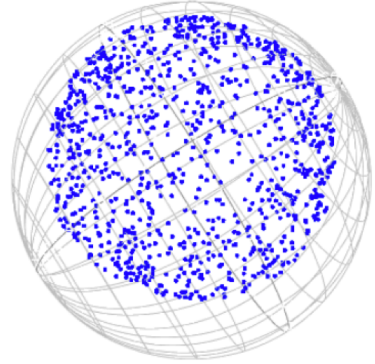
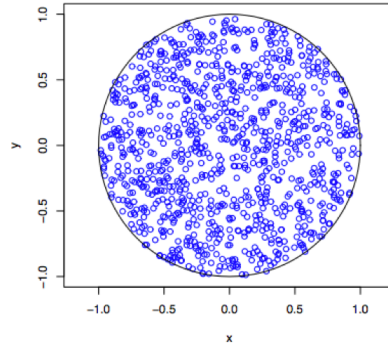
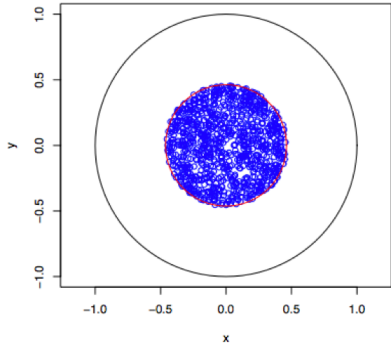
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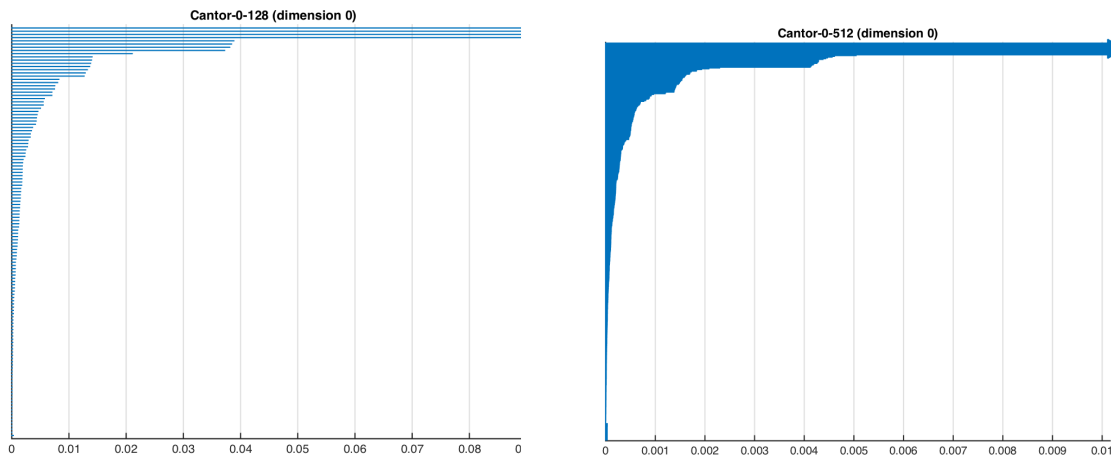
Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017

# Local geometry



Persistent homology detects curvature  
Bubenik, Hull, Patel, Whittle, 2019

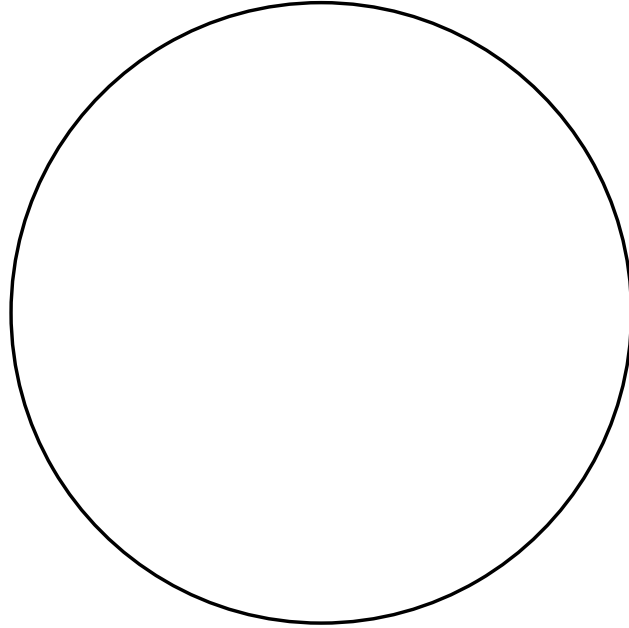
# Local geometry



A fractal dimension for measures via persistent homology  
Adams, Aminian, Farnell, Kirby, Peterson, Mirth,  
Neville, Shonkwiler, 2020

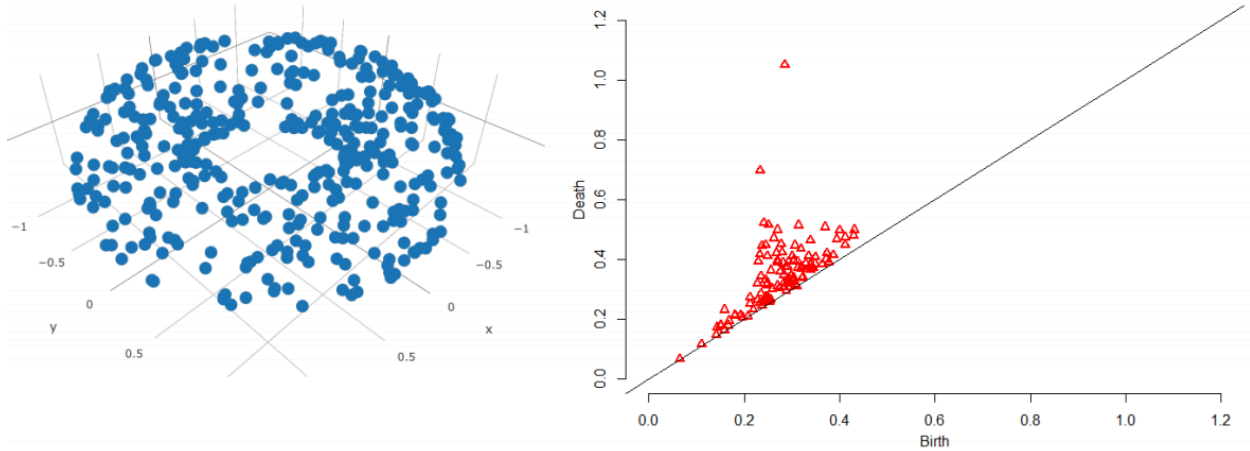


## Local geometry



A fractal dimension for measures via persistent homology  
Adams, Aminian, Farnell, Kirby, Peterson, Mirth,  
Neville, Shonkwiler, 2020

# Local geometry



On the choice of weight functions for linear representations of persistence diagrams  
Divol and Polonik, 2019

# From persistent homology to machine learning



Persistent homology measures both the global topology and the local geometry of a dataset.