Applied Topology: From Global to Local

Henry Adams
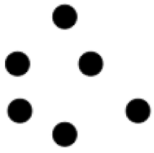## Colorado State University

Joint with Michael Moy

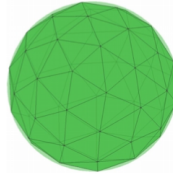# From persistent homology to machine learning

Persistent homology measures both the global topology and the local geometry of a dataset.
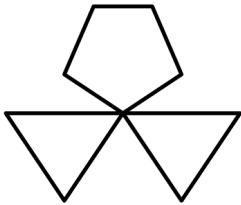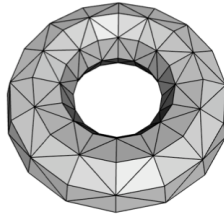
Global topology

H_0: rank 6
H_1: rank 0
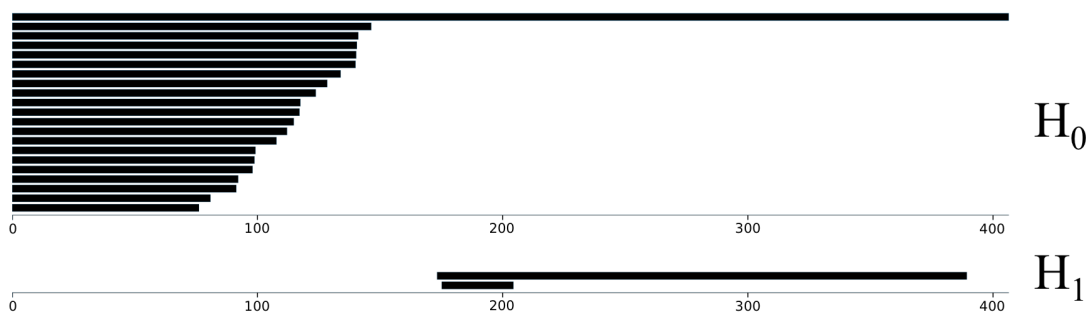

H_0: rank 1
H_1: rank 0
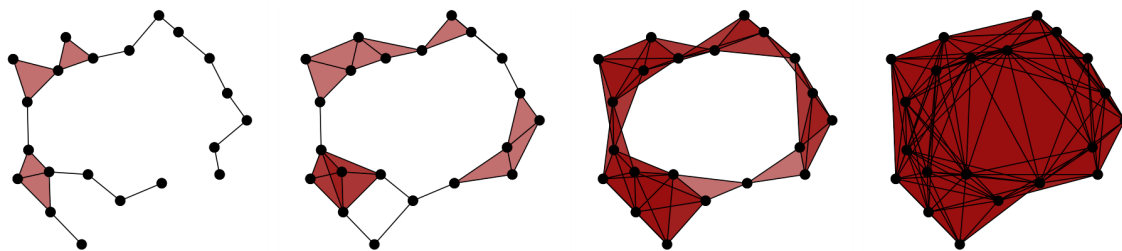H_2: rank 1
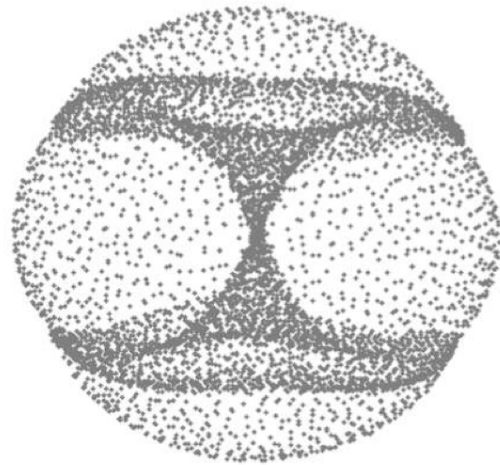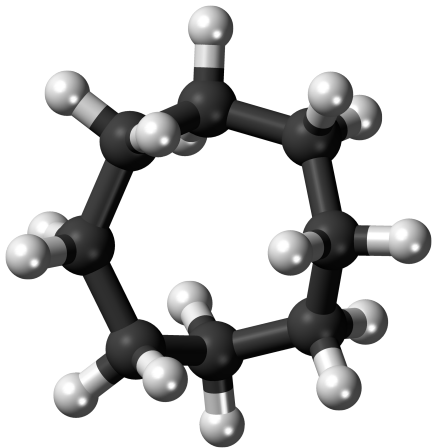

H_0: rank 1
H_1: rank 3

H_0: rank 1
H_1: rank 2
H_2: rank 1
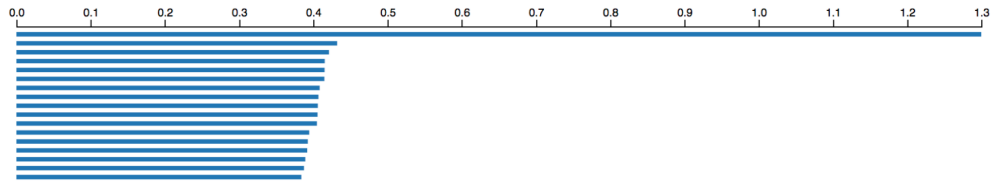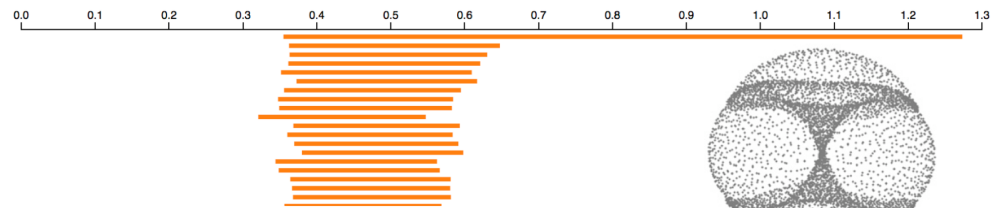
Global topology

Global topology

Topology of cyclo-octane energy landscape
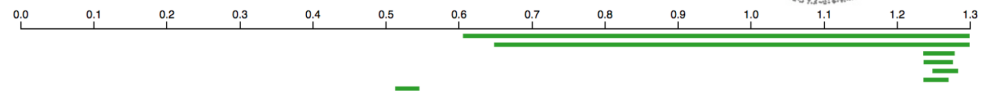Martin, Thompson, Coutsias, Watson, 2010

Global topology

Persistence intervals in dimension 0:

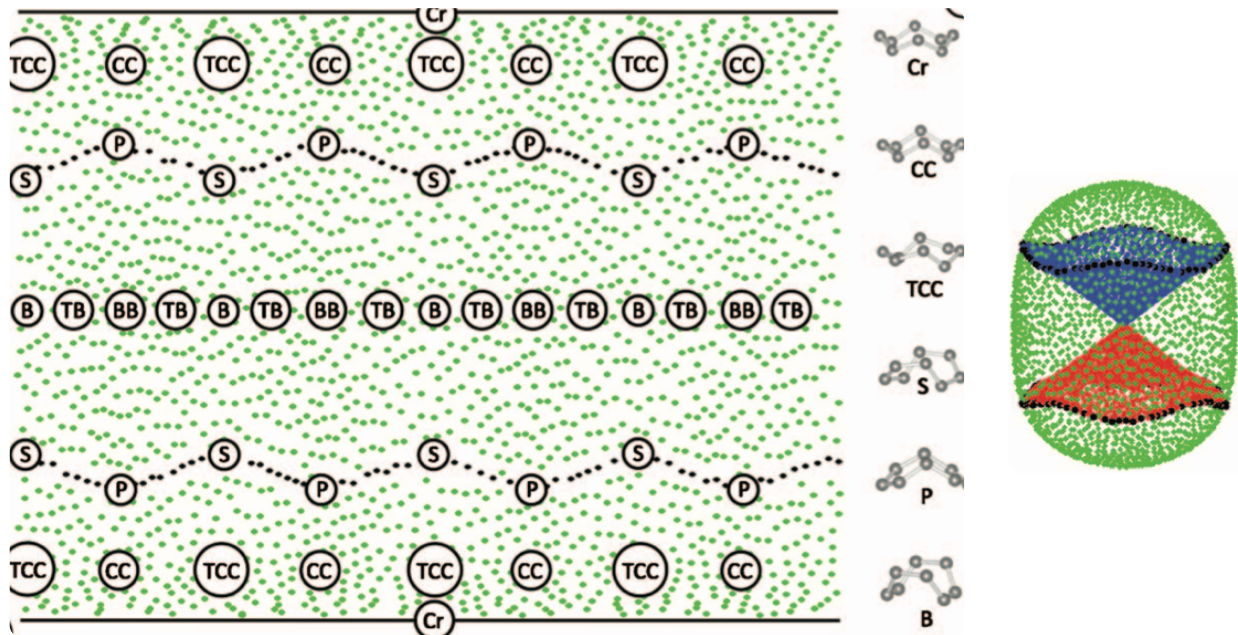Persistence intervals in dimension 1:

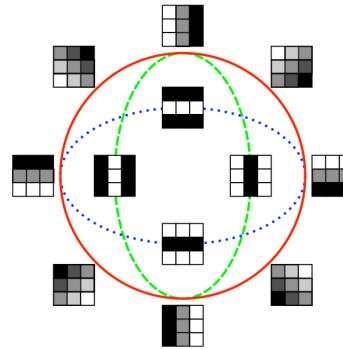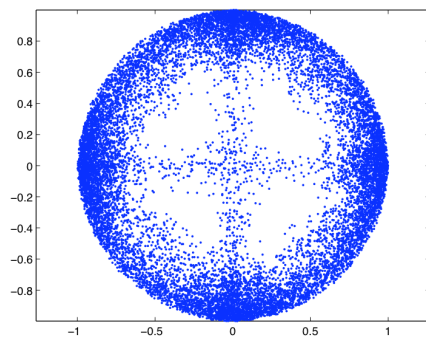Persistence intervals in dimension 2:

Topology of cyclo-octane energy landscape
Martin, Thompson, Coutsias, Watson, 2010
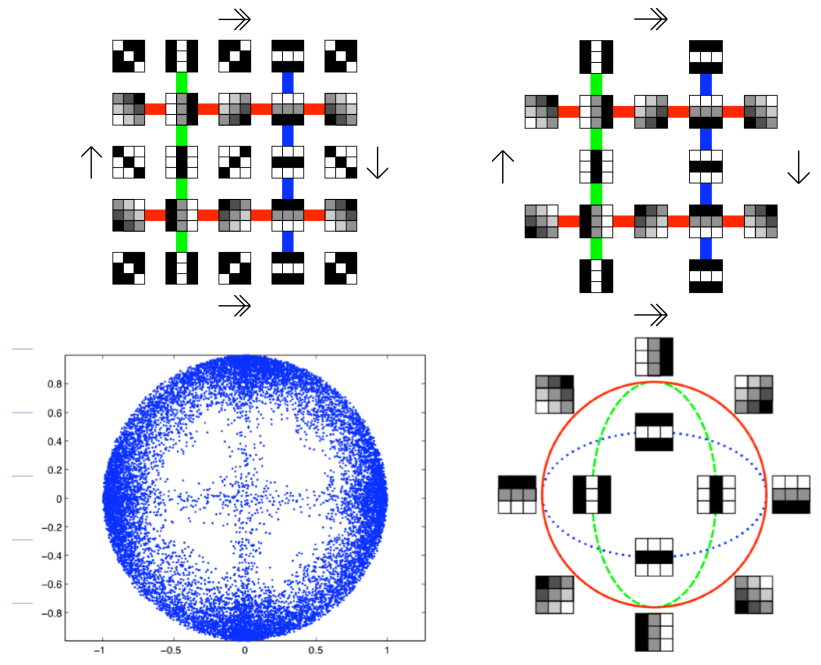
Global topology

Topology of cyclo-octane energy landscape
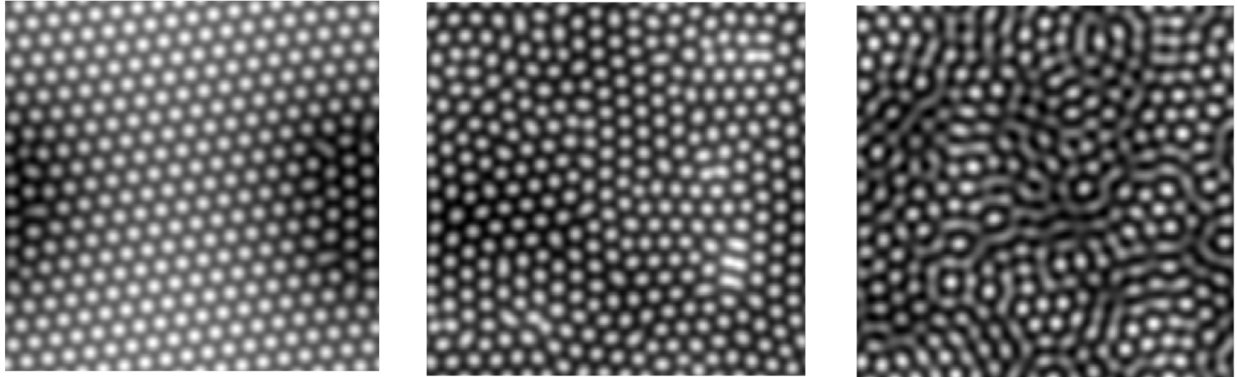Martin, Thompson, Coutsias, Watson, 2010

Global topology

On the local behavior of natural images
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008
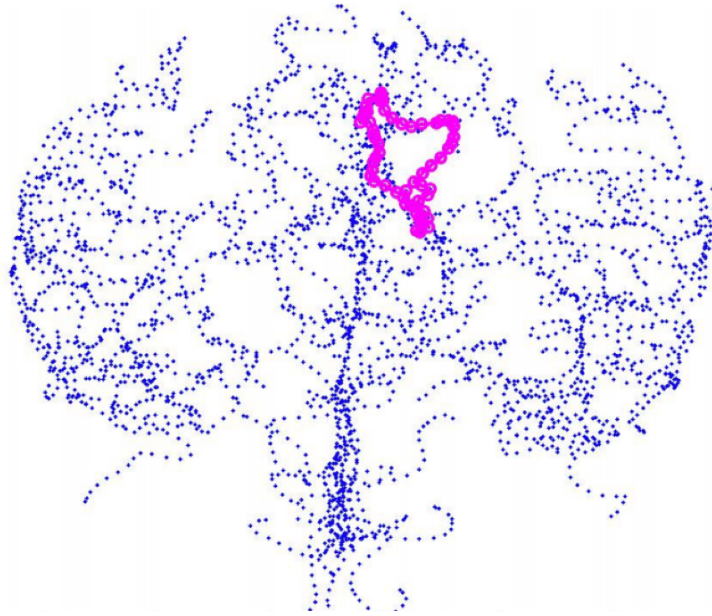
Global topology

On the local behavior of natural images
Carlsson, Ishkhanov, de Silva, Zomorodian, 2008
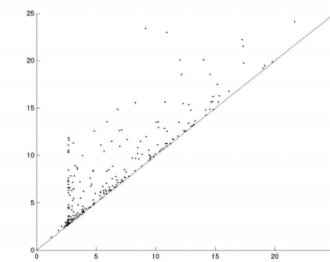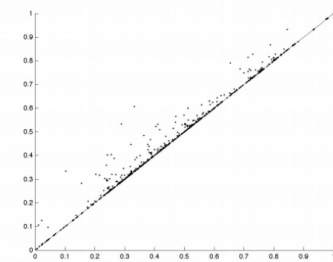
Local geometry
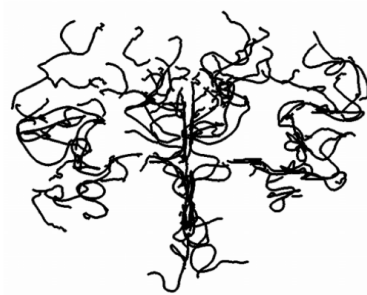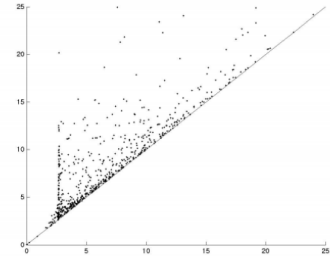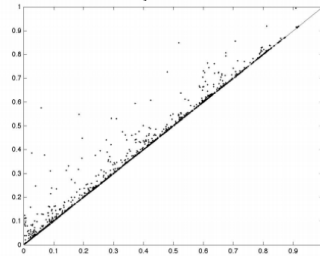

Measures of order for nearly hexagonal lattices
Motta, Neville, Shipman, Pearson, Bradley, 2018

Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skwerer, 2014

Local geometry

Persistent homology analysis of brain artery trees
Bendich, Marron, Miller, Pieloch, Skwerer, 2014

Local geometry

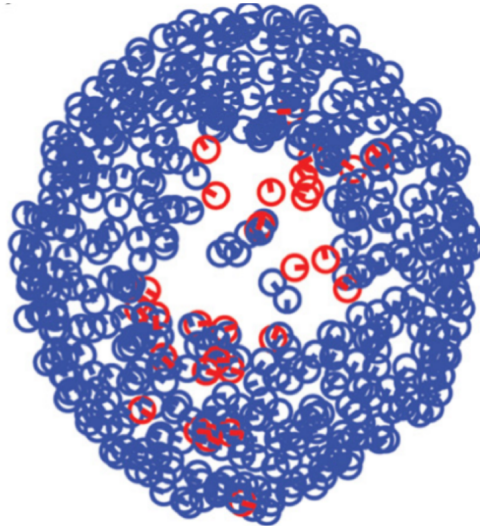Collective motion, self-organization

Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
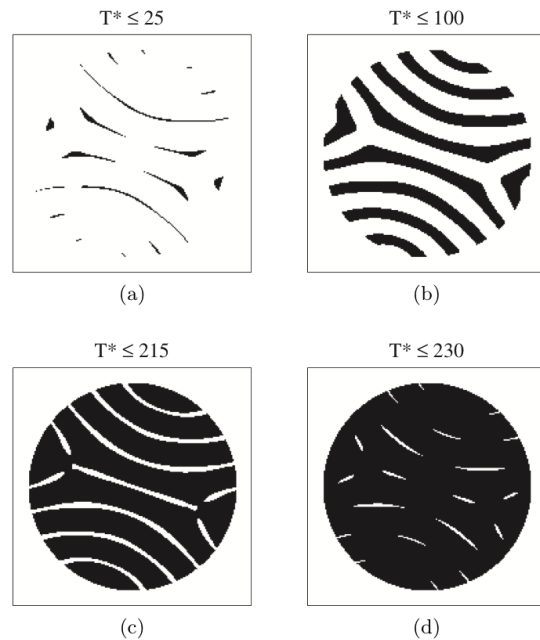Topaz, Ziegelmeier, Halverson, 2015
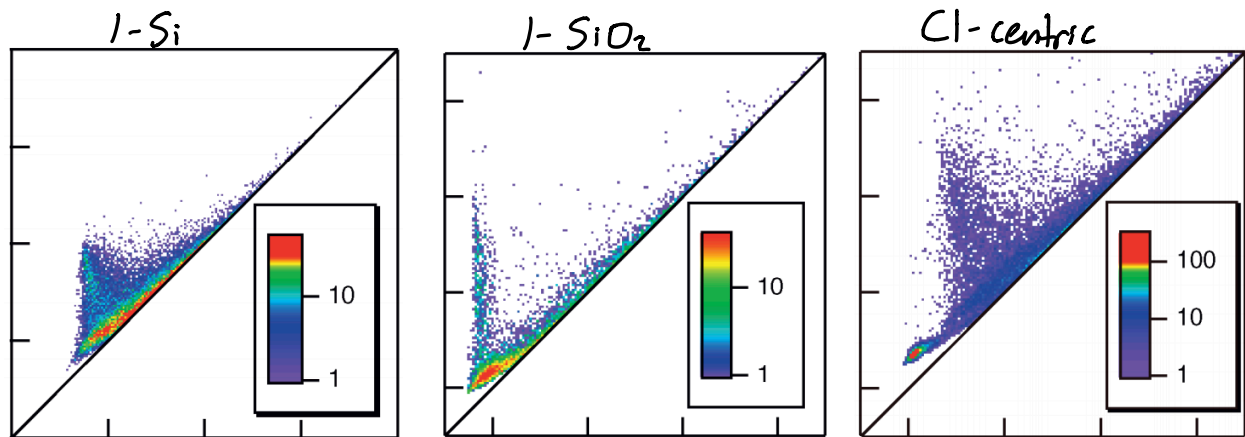
Local geometry

Collective motion, self-organization

Topological data analysis of biological aggregation models
Topaz, Ziegelmeier, Halverson, 2015

Local geometry

T* ≤ 25   T* ≤ 100

(a)   (b)

T* ≤ 215   T* ≤ 230

(c)   (d)

Analysis of Kolmogorov flow and Rayleigh-Bénard convection using persistent homology
Kramár, Levanger, Tithof, Suri, Xu, Paul, Schatz, Mischaikow, 2016

Local geometry

Understanding diffusion patterns of glassy, liquid and amorphous materials via persistent homology analysis
Onodera, Kohara, Tahara, Masuno, Inoue, Shiga, Hirata, Tsuchiya, Hiraoka, Obayashi, Ohara, Mizuno, Sakata, 2019
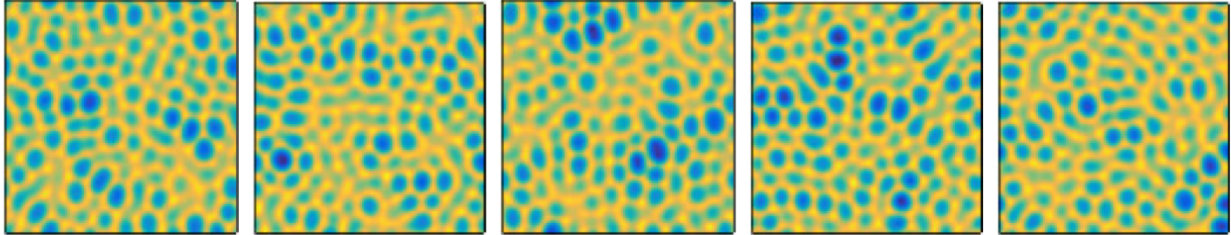
Local geometry
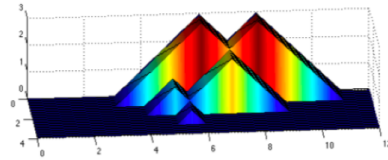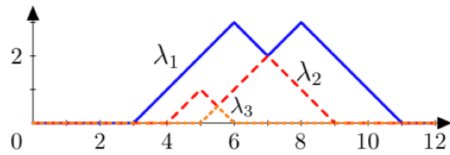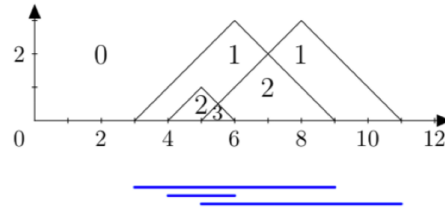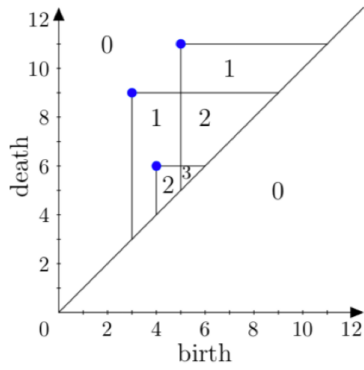
Statistical topological data analysis using persistence landscapes
Bubenik, 2015

Local geometry
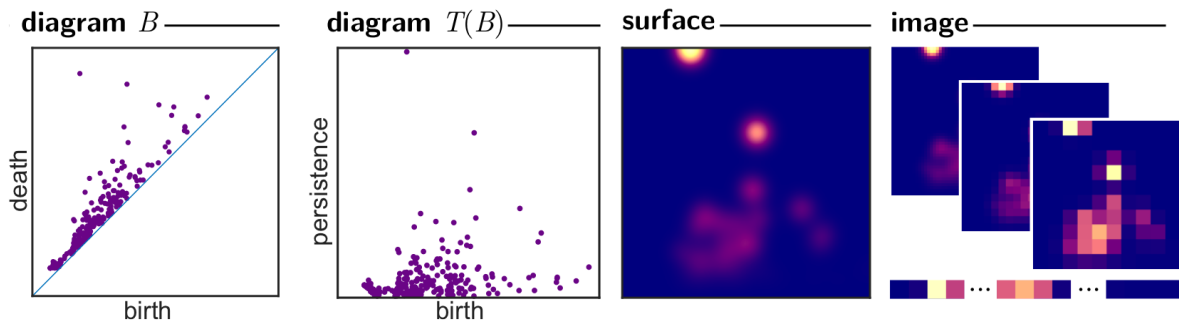
diagram B — death / birth
diagram $T(B)$ — persistence / birth
surface —
image —

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017

Local geometry

Different parameters:

Same parameter:

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017
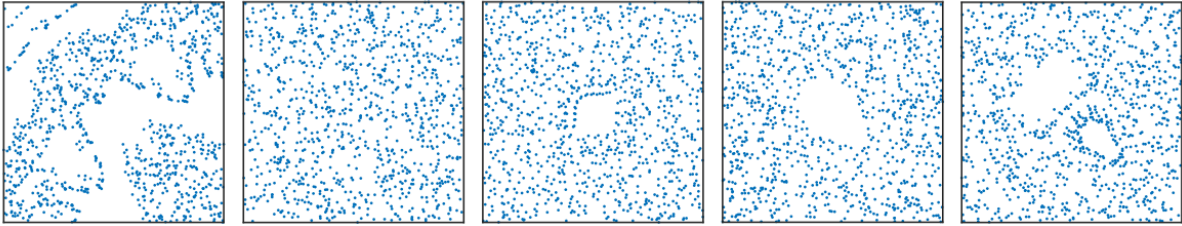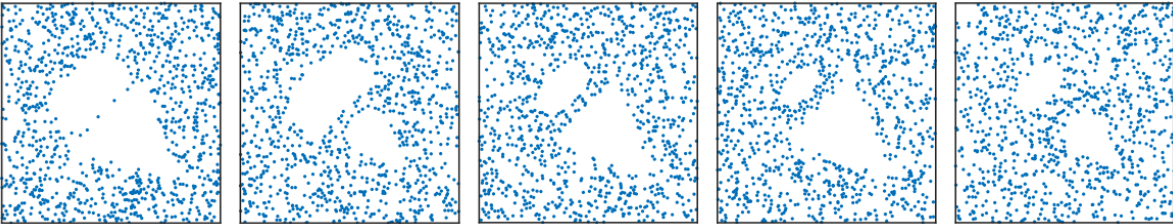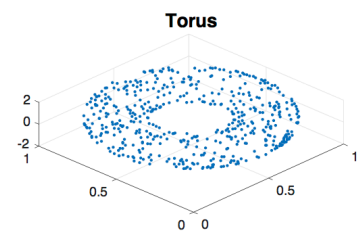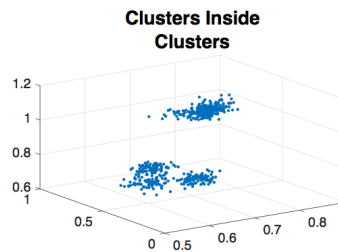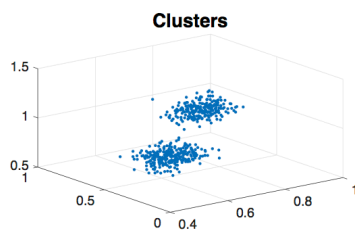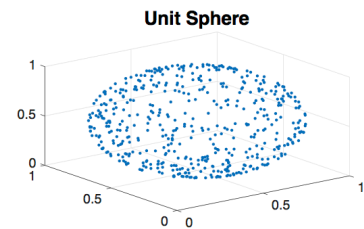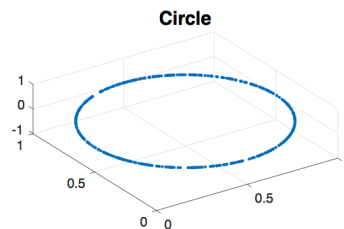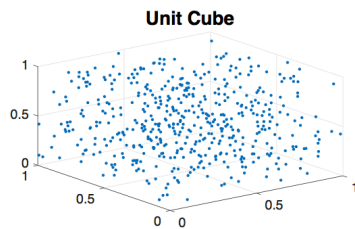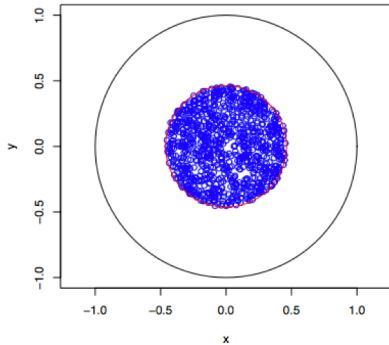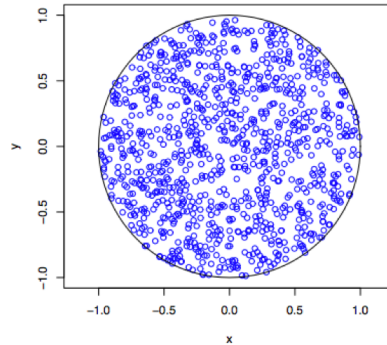
Local geometry

Persistence images: A stable vector representation of persistent homology. Adams, Chepushtanova, Emerson, Hanson, Kirby, Motta, Neville, Peterson, Shipman, Ziegelmeier, 2017

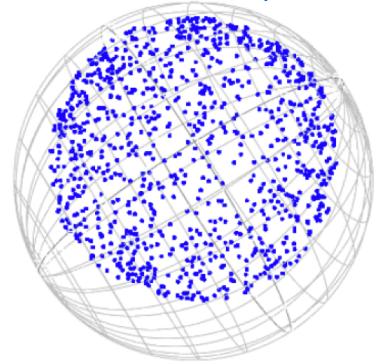# Local geometry

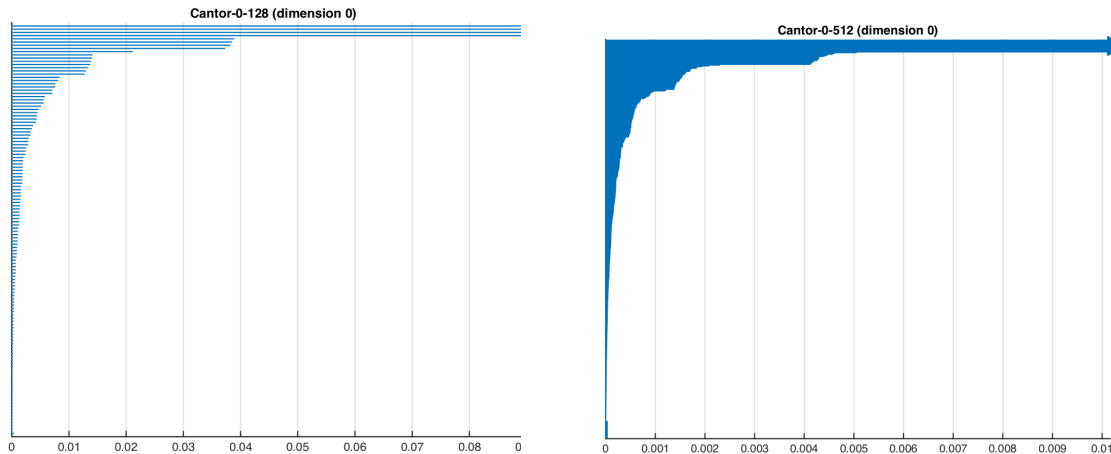### Hyperbolic disk   ### Flat disk   ### Disk on sphere

Persistent homology detects curvature
Bubenik, Hull, Patel, Whittle, 2019

Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth,
Neville, Shonkwiler, 2020
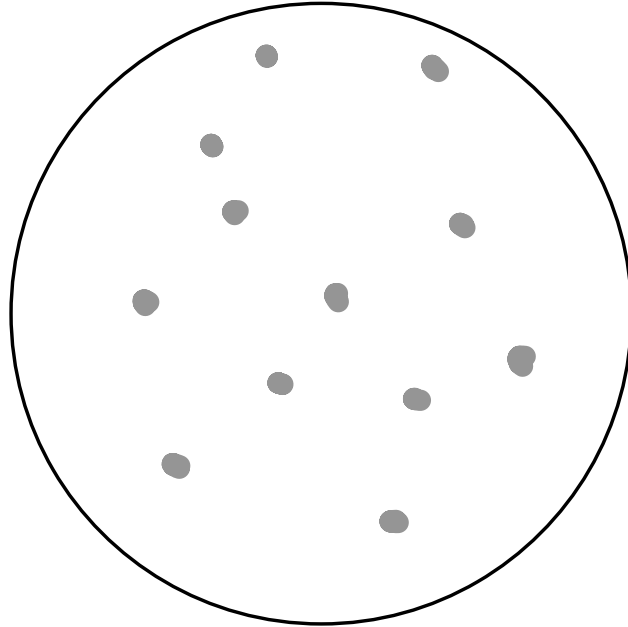
See also work by Robins and MacPherson & Schweinhart

Local geometry

A fractal dimension for measures via persistent homology
Adams, Aminian, Farnell, Kirby, Peterson, Mirth,
Neville, Shonkwiler, 2020

Local geometry

A fractal dimension for measures via persistent homology
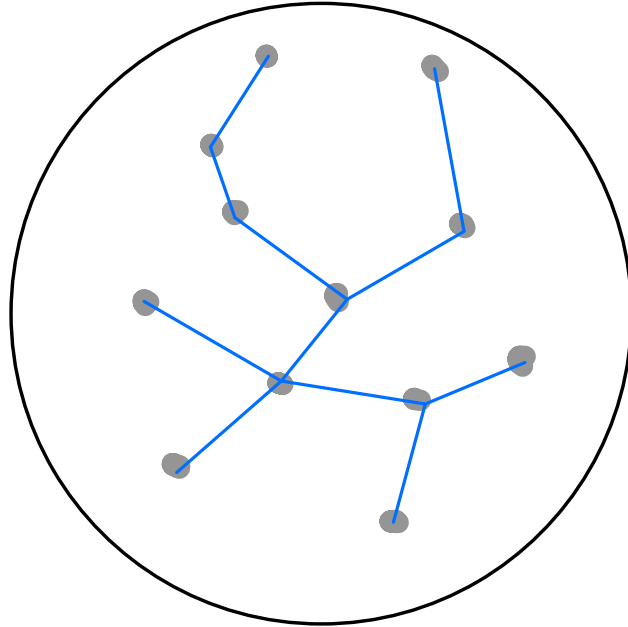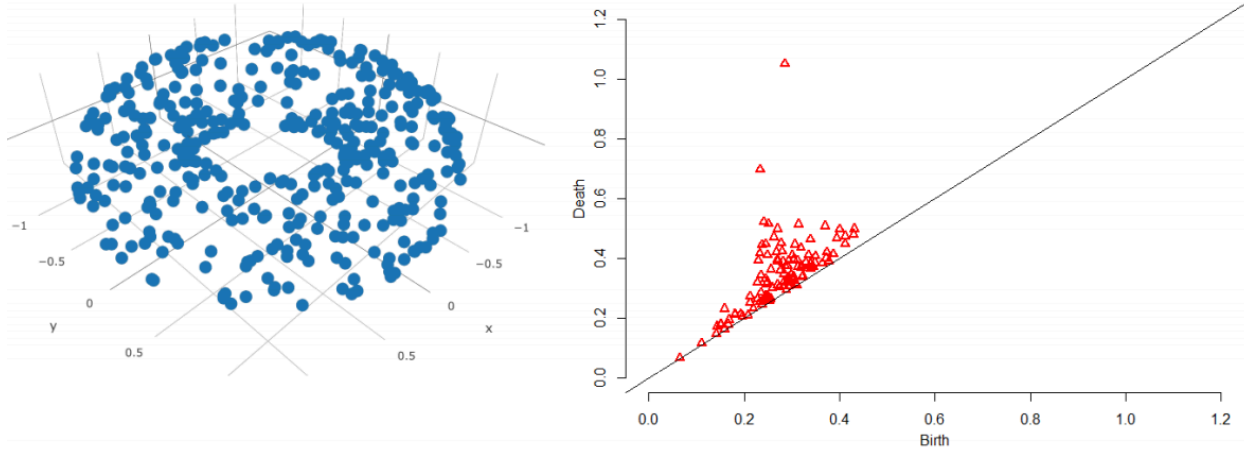Adams, Aminian, Farnell, Kirby, Peterson, Mirth,
Neville, Shonkwiler, 2020

Local geometry

On the choice of weight functions for linear
representations of persistence diagrams
Divol and Polonik, 2019

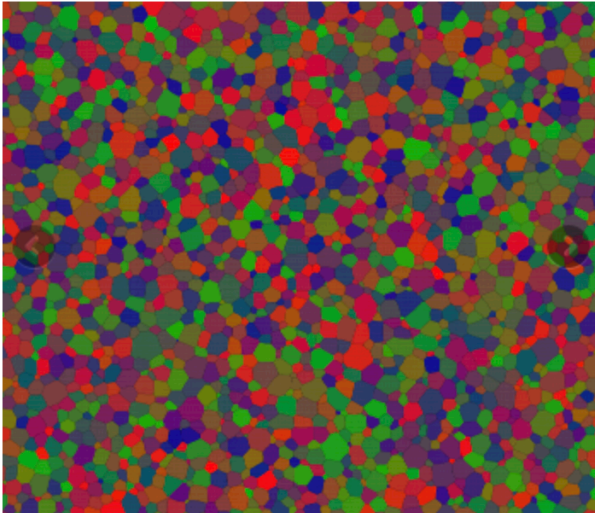# Applied Topology: From Global to Local

Henry Adams
Colorado State University

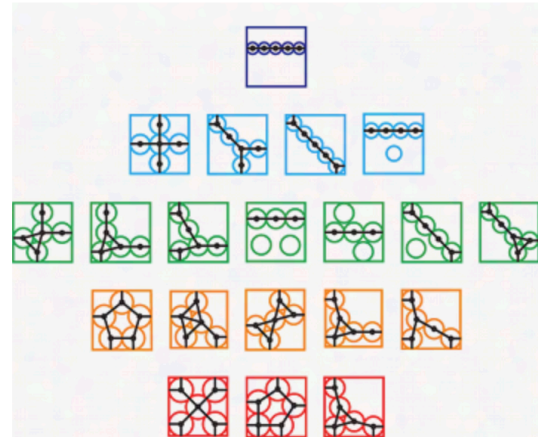# From persistent homology to machine learning

Persistent homology measures both the global topology and the local geometry of a dataset.
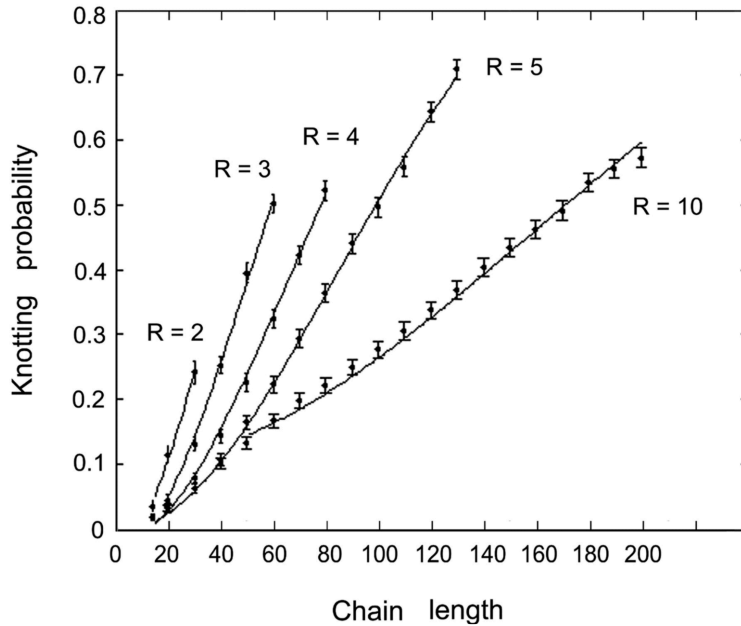
Local geometry

Cellular automata simulation of grain growth.

Jeremy Mason research group @UC Davis

Local geometry

Knotting probabilities of DNA molecules confined in restricted volumes: DNA knotting in phage capsids
Arsuaga, Vázquez, Trigueros, Sumners, Roca, 2002