Research Statement: The behavior of geometric complexes

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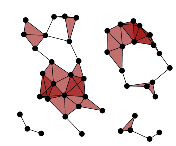
As the demands for ways to collect, process, analyze, and understand complex datasets continue to grow, techniques summarizing the "shape" of a dataset are needed. Indeed, the shape of a dataset often reflects important patterns within, whose discovery allows one to better store, model, and make inferences from the data. One of the main algorithms in applied topology, persistent homology [11, 12], provides insight into the shape of nonlinear, high-dimensional, time-varying, and noisy datasets at multiple different scales. My work revolves around persistent homology, from both the theoretical and applied perspectives.

Given a finite sample of data points near an unknown underlying shape, what information about the unknown shape can be recovered from the sample? When the ambient dimension is low, there are effective techniques for building a mesh on top of the sample. However, the computation required for meshing scales poorly as the ambient dimension grows; this is one example of the "curse of dimensionality." Even data from our 3D world can quickly become high-dimensional: a 100×100 pixel greyscale image can be thought of as a point in 10,000 dimensions. In high dimensions, a more common approach than meshing is to instead build a Vietoris–Rips complex, due to its computational feasibility. The Vietoris–Rips complex contains as its simplices all subsets of a bounded diameter; it "thickens" the sample to approximate the unknown underlying space.



Under certain assumptions (the chosen scale parameter is sufficiently small when compared to the curvature), one can guarantee that the Vietoris–Rips complex of a dataset recovers the shape of the unknown underlying space. Since one does not know a priori how to choose the scale, the idea of persistent homology is to compute the Vietoris–Rips complex over a range of scales, and to trust those topological features which persist as the scale is varied.

Little is known about Vietoris–Rips complexes at larger scales, though they arise naturally in applications of persistent homology. My work extends this theory to larger scales. In [1, 3] we prove that the Vietoris–Rips complexes of the circle are homotopy equivalent to the circle, 3-sphere, 5-sphere, 7-sphere, . . . , as the scale increases. This is the first connected non-contractible Riemannian manifold whose Vietoris–Rips homotopy types are known at all scales, providing a foothold towards deeper understanding. In [2], we use optimal transport [17, 13] to redefine the (often non-metrizable) Vietoris–Rips complex instead as a metric space, allowing us to identify Vietoris–Rips thickenings of n-spheres. In future work I am developing a Morse theory for infinite-dimensional thickenings: one will be able to understand the Vietoris–Rips thickenings of a shape without inventing new tools for each new shape.

Both mathematics and its applications are strengthened when the two are studied simultaneously. I have worked on applications of Vietoris–Rips complexes to machine learning [9], sensor networks [8, 5], computer vision [7], and high-dimensional data analysis [6]. Many of my research projects are with undergraduates [4, 5, 10, 14, 15]: I find that students whose work overlaps with applications are strongly motivated to learn, use, and improve the underlying mathematics. I am also a coauthor of the tutorial for the persistent homology software Javaplex [16], written for non-mathematicians and equipped with exercises, solutions, and examples on real life data. As the Director of the Applied Algebraic Topology Research Network (AATRN), with over 400 members, I run an online research seminar which shares our tools with the broader scientific community. Our YouTube channel has had 24,000 minutes watched in the past year, and we are working hard to continue to grow.

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