Homework 8
Due: Friday, March 27

1. Suppose $\mathcal{F}$ is a coherent sheaf on X and $P \in X$. Suppose that the stalk $\mathcal{F}_P$ is a free $\mathcal{O}_{X,P}$-module. Show that there is an open neighborhood U of P such that $\mathcal{F}|_U$ is free.

   (HINT: *Suppose $\mathcal{F}_P = \oplus \mathcal{O}_{X,P} f_i$. Find an open neighborhood V and a candidate $\mathcal{O}_X(U)$-module M. Compare $\widetilde{M}$ to $\mathcal{F}|_U$...*)

2. Let k be a field, $\lambda \in k - \{0,1\}$. Let $E_\lambda = \operatorname{Proj} k[X,Y,Z]/(Y^2Z - X(X-Z)(X-\lambda Z))$, i.e., E_λ is
$$\mathcal{Z}_{\mathbb{P}^2}(Y^2Z - X(X-Z)(X-\lambda Z)).$$

   It turns out that E_λ can also be described as the vanishing locus of the quadrics $XW - YZ$ and $YW - (X-Z)(X-\lambda Z)$ in $\mathbb{P}^3$.

   Let $i : E_\lambda \to \mathbb{P}^2$ be the first embedding, and let $j : E_\lambda \to \mathbb{P}^3$be the second embedding.

   Show that $\mathcal{F} = i^*\mathcal{O}_{\mathbb{P}^2}(1)$ is not the same line bundle as $\mathcal{G} = j^*\mathcal{O}_{\mathbb{P}^2}(1)$.

   (HINT: *If you like, let s be a nonzero element $\mathcal{F}(E_\lambda)$, and let t be a nonzero element of $\mathcal{G}(E_\lambda)$. Using techniques from last semester, show that $E_\lambda - E_{\lambda,s}$ typically consists of three points, while $E_\lambda - E_{\lambda,t}$ typically consists of four points.*)