1. Let $A$ be a ring. For each $r, n \in \mathbb{Z}$, calculate $H^r(\mathbb{P}^r_A, \mathcal{O}_{\mathbb{P}^r_A}(n))$. (HINT: Use the standard cover of $\mathbb{P}^1$ as the union of two affine lines.)

2. Let $F(X_0, X_1, X_2) \in k[X_0, X_1, X_2]$ be homogeneous of degree $d$, and consider the curve $X = Z_+(X) \subset \mathbb{P}^2_k$. Suppose $[1, 0, 0] \notin X$.

   Let $U_1 = X \cap \{X_1 \neq 0\}$ and let $U_2 = X \cap \{X_2 \neq 0\}$.

   (a) Show that $U = \{U_1, U_2\}$ is an open cover of $X$.

   (b) Use the Cech complex $\mathcal{C}^\bullet(U, \mathcal{O}_X)$ to calculate the cohomology groups $H^\bullet(X, \mathcal{O}_X)$ explicitly. (HINT: You should find that $\dim_k H^0(X, \mathcal{O}_X) = 1$ and $\dim_k H^1(X, \mathcal{O}_X) = \frac{(d-1)(d-2)}{2}$.)

3. For a scheme $X$, let $\mathcal{O}_X^\times$ be the sheaf of abelian groups $U \mapsto \mathcal{O}_X(U)^\times$, which assigns to an open set $U$ the group of invertible functions on $U$.

   Suppose $X$ is separated and quasicompact.

   (a) Let $\mathcal{L}$ be a quasicoherent sheaf on $X$. In class, we said that $\mathcal{L}$ is invertible if and only if:

      There exists a cover $U = \bigcup U_i$ of $X$, and elements $g_{ij} \in \mathcal{O}_X(U_{ij})^\times$, such that $g_{jk} \cdot g_{ij} = g_{ik} \in \mathcal{O}_X(U_{ijk})^\times$. Make sure you understand this.

   (b) For $\mathcal{L}$ and $U$ as above, explain how to construct an element $\phi_U(\mathcal{L})$ of $H^1_U(X, \mathcal{O}_X^\times)$, and thus an element $\phi(\mathcal{L}) \in H^1(X, \mathcal{O}_X^\times)$. Show that invertible sheaves on $X$ gives an isomorphism

   \[ \begin{array}{ccc} \mathcal{L} & \xrightarrow{\phi} & H^1(X, \mathcal{O}_X^\times) \\ \phi(\mathcal{L}) & \end{array} \]

   is a group homomorphism. What is $\ker \phi$?

   (c) Show that $\phi$ induces an isomorphism

   \[ \text{Pic}(X) \xrightarrow{\phi} H^1(X, \mathcal{O}_X^\times) \]

   (Remember, any element of $H^1(X, \mathcal{O}_X^\times)$ can be represented by a Cech cocycle on some open cover of $X$.)

   (d) Read Liu exercise 5.2.7, especially 5.2.7.d.

4. (a) Liu 5.2.17.a. Show that the $d$ defined here satisfies $d \circ d = 0$. Note: It suffices to assume that $U = V$; why?

   (b) Liu 5.2.17.b.