## Homework 7 Due: Friday, March 9

- 1. Liu, 4.2.7. (HINT: See last semester, HW12#1)
- 2. Liu, 4.2.3. Ignore normality.
- 3. Let *A* be a *B*-algebra; we identify *B* with its image in *A*. Consider the homomorphism

$$A \otimes_B A \xrightarrow{f} A$$
$$a_1 \otimes a_2 \longmapsto a_1 a_2$$

(This corresponds to  $\Delta$  : Spec  $A \rightarrow$  Spec  $A \times_{\text{Spec } B}$  Spec A.) Its kernel I = ker(f) is generated by elements of the form  $a \otimes 1 - 1 \otimes a$ .

Think of  $A \otimes_B A$  as an A-module via left multiplication. Consider the map of A-modules

$$A \xrightarrow{\widetilde{d}} A \otimes_B A$$

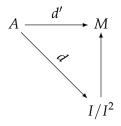
$$a \longmapsto 1 \otimes a - a \otimes 1.$$

- (a) Show that  $\widetilde{d}(A) \subseteq I \subseteq A \otimes_B A$ .
- (b) Show that  $\tilde{d}$  is additive;  $\tilde{d}(a_1 + a_2) = \tilde{d}(a_1) + \tilde{d}(a_2)$ .
- (c) Show that  $\tilde{d}(B) = 0$ .
- (d) Show that  $\tilde{d}(a_1a_2) (a_1\tilde{d}(a_2) + a_2\tilde{d}(a_1)) \in I^2$ .
- (e) Conclude that the composition *d* :

$$A \longrightarrow A \otimes_B A \longrightarrow I/I^2$$

is an element of  $\text{Der}_B(A, I/I^2)$ , the set of derivations from *A* to  $I/I^2$  which vanish on *B*. (See 10.1.2 from last semester for the definition of a derivation in the special case B = k.)

*Remark:* In fact, this is a universal derivation, in the sense that if M is any A-module, and if  $d': A \rightarrow M$  is a B-linear derivation, then d' factors uniquely:



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