Homework 6

Due: Friday, March 2

1. Let $f : X \to Y$ be a morphism.

   The morphism is called *finite* if for every open affine subset $V = \operatorname{Spec} B \subseteq Y$, the inverse image $f^{-1}(V) = U = \operatorname{Spec} A$ is affine, with A a finite B-module. (See also Liu, exercise 3.3.15, for equivalent formulations.)

   The morphism is called *quasifinite* if for each $Q \in Y$, $f^{-1}(Q)$ is a finite set.

   (a) Show that a finite morphism is quasifinite.

   (b) Given an example of a morphism which is surjective, of finite type, and quasifinite but not finite.

2. Suppose Y is irreducible, with generic point η_Y. A morphism $f : X \to Y$ is *generically finite* if $f^{-1}(\eta_Y)$ is a finite set.

   (a) Suppose X and Y are both integral schemes. Let $f : X \to Y$ be a generically finite morphism of finite type. Show that the function field $k(X) := \mathcal{O}_{X,\eta_X}$ is a finite extension of $k(Y)$.

   (b) Show that there is an open dense subset $U \subseteq Y$ such that the induced morphism $f^{-1}(U) \to U$ is finite. (HINT: *Use the result of Liu, exercise 3.2.16.b*)

3. Liu 3.3.5. (HINT: *Show that the image of X in Y is both open and closed.*)

4. (a) Liu 3.3.6. (HINT: *For* section *see 2.3.28.*)

   (b) Liu 3.3.10.a

5. Use the valuative criterion to show that $\mathbb{A}^1_k \to \operatorname{Spec} k$ is not proper.

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