Homework 3
Due: Friday, February 9

1. Let $X$ be the affine scheme
   \[ X = \text{Spec} \left( \frac{\mathbb{Z}[x, y, z]}{(x^2 + y^2 - z^2)} \right). \]
   Let $K$ be any field. Carefully explain the bijection between:
   - Maps of schemes $\text{Spec } K \to X$; and
   - Triples $\alpha, \beta, \gamma \in K$ such that $\alpha^2 + \beta^2 = \gamma^2$.

   This should explain why $\text{Mor}(\text{Spec } K, X)$ is called the set of $K$-points of $X$; it is often written as $X(\text{Spec } K)$, or even $X(K)$.

2. Liu, 2.3.7.

3. Let $R = \mathbb{Z}$, and let $S = \mathbb{Z}[x, y]$ with the usual grading. Consider the set $D_+((3)) \subseteq \text{Proj}(S)$ and the point $P = (7, x^2 + y^2) \in \text{Proj}(S)$.
   (a) Show that $(7, x^2 + y^2)$ really is a prime ideal of $\mathbb{Z}[x, y]$ which doesn’t contain $(x, y)$.
   (b) Find an element $f \in \mathbb{Z}[x, y]$ such that $f \in S_+$ is homogeneous and
   \[ P \in D_+(f) \subseteq D_+(3). \]