Homework 3
Due: Friday, February 9

1. Let X be the affine scheme
$$X = \operatorname{Spec} \frac{\mathbb{Z}[x,y,z]}{(x^2+y^2-z^2)}.$$
Let K be any field. Carefully explain the bijection between:
   - Maps of schemes $\operatorname{Spec} K \to X$; and
   - Triples $\alpha, \beta, \gamma \in K$ such that $\alpha^2 + \beta^2 = \gamma^2$.

   This should explain why $\operatorname{Mor}(\operatorname{Spec} K, X)$ *is called the set of* K*-points of* X*; it is often written as* $X(\operatorname{Spec} K)$*, or even* $X(K)$*.*

2. Liu, 2.3.7.

3. Let $R = \mathbb{Z}$, and let $S = \mathbb{Z}[x,y]$ with the usual grading. Consider the set $D_+((3)) \subseteq \operatorname{Proj}(S)$ and the point $P = (7, x^2+y^2) \in \operatorname{Proj}(S)$.
   (a) Show that $(7, x^2+y^2)$ really is a prime ideal of $\mathbb{Z}[x,y]$ which doesn't contain (x,y).
   (b) Find an element $f \in \mathbb{Z}[x,y]$ such that $f \in S_+$ is homogeneous and
$$P \in D_+(f) \subset D_+(3).$$