Let  $A = k[x, y]/(y^2 - x^2(x - 1))$ , and let X = Spec A. Let  $\mathfrak{p} = (x, y)$ , let  $\mathfrak{q} = (x - 1, y)$ , and let P and Q be the corresponding points. Recall that localization commutes with taking quotients. Then:

$$\mathcal{O}_{X,P} = A_{\mathfrak{p}}$$
  

$$\mathfrak{m}_{P} = (x, y)A_{\mathfrak{p}}$$
  

$$\mathfrak{m}_{P}/\mathfrak{m}_{P}^{2} = \frac{(x, y)\left(\frac{k[x,y]}{(y^{2}-x^{2}(x-1))}\right)_{(x,y)}}{(x^{2}, xy, y^{2})\left(\frac{k[x,y]}{(y^{2}-x^{2}(x-1))}\right)_{(x,y)}}$$
  

$$\cong \frac{(x, y)k[x, y]_{(x,y)}}{(x^{2}, xy, y^{2}, y^{2} - x^{2}(x-1))}$$
  

$$\cong kx \oplus ky \text{ as } \kappa(P) = k \text{ module}$$

while

$$\begin{aligned} \mathcal{O}_{X,Q} &= A_{\mathfrak{q}} \\ \mathfrak{m}_{Q} &= (x-1,y)A_{\mathfrak{q}} \\ \mathfrak{m}_{Q}/\mathfrak{m}_{Q}^{2} &= \frac{(x-1,y)\left(\frac{k[x,y]}{(y^{2}-x^{2}(x-1))}\right)_{(x-1,y)}}{((x-1)^{2},(x-1)y,y^{2})\left(\frac{k[x,y]}{(y^{2}-x^{2}(x-1))}\right)_{(x-1,y)}} \\ &\cong \frac{(x-1,y)k[x,y]_{(x-1,y)}}{((x-1)^{2},(x-1)y,y^{2},y^{2}-x^{2}(x-1))} \end{aligned}$$

But  $x^2$  is a unit after localization at (x - 1, y), thus

$$\cong \frac{(x-1,y)k[x,y]}{(x-1)^2,(x-1)Y,y^2,(x-1)}$$
$$\cong ky$$

Take  $k = \mathbb{R}$ , and graph *X*, *P* and *Q*.

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