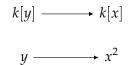
Homework 6 Due: Friday, March 3

1. [GW] 4.15.

2. Let $X = \operatorname{Spec} \mathbb{Z}[x]/(x^2+1)$ and $Y = \operatorname{Spec} \mathbb{Z}$; consider the map $X \to Y$ which comes from $\mathbb{Z} \hookrightarrow \mathbb{Z}[x]/(x^2+1)$.

Suppose $Q \in Y$. Describe the fiber X_Q . (HINT: Your answer will depend on Q = [(q)].)

- 3. (a) Describe Spec $\mathbb{C} \times_{\text{Spec } \mathbb{R}}$ Spec \mathbb{C} .
 - (b) Let $X = \operatorname{Spec} k[x]$, $Y = \operatorname{Spec} k[y]$, and let ϕ be the morphism $X \to Y$ attached to



- i. Suppose char(k) \neq 2. Show that $X \times_Y X$ has two irreducible components.
- ii. Suppose char(k) = 2. Describe $X \times_Y X$.

(HINT: It may be marginally easier to think of two copies of X, X_1 and X_2 , with coordinates x_1 and x_2 ...)

4. Weil restriction Let $T \to S$ and $X \to T$ be schemes. The Weil restriction $\mathbf{R}_{T/S}(X)$ is the scheme such that, for each *S*-scheme $Z \to S$,

$$\mathbf{R}_{T/S}(X)(Z) = \operatorname{Mor}_{(} T \times_{S} Z, X).$$

(If T = Spec B and S = Spec A, this is usually just written as $\mathbf{R}_{B/A}(X)$.) Let $\mathbb{G}_m = \text{Spec } \mathbb{R}[s,t]/(st-1) \cong \text{Spec } \mathbb{R}[s,s^{-1}]$.

- (a) Show that G_m represents the functor on \mathbb{R} -schemes $Z \to \mathcal{O}_Z(Z)^{\times}$ (the units in $\mathcal{O}_Z(Z)$).
- (b) Show that

$$\mathbf{R}_{\mathbb{C}/\mathbb{R}}(\mathbb{G}_m) \cong \operatorname{Spec}(\mathbb{R}[x, y, \frac{1}{x^2 + y^2}]).$$

(Of course, $\mathbb{R}[x, y, (x^2 + y^2)^{-1}]$ means $\mathbb{R}[x, y, z] / (z(x^2 + y^2) - 1)$.)

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