
Homework 2

Due: Friday, February 3

[GW] refers to problems in the text Görtz and Wedhorn, *Algebraic Geometry I*.

1. [GW] 2.2. *Try to prove this directly, rather than using something like Prop. 2.3.*

2. [GW]2.8(b),(c).

3. Let X be a set with two elements, equipped with the discrete topology; so, the open subsets of X are X; two one-point sets U and V; and $\emptyset$.

   Consider the following presheaf $\mathcal{F}$ of abelian groups on X:

   $$\begin{aligned} \mathcal{F}(X) &= \mathbb{Z} \oplus \mathbb{Z} \\ \mathcal{F}(U) &= \mathbb{Z}/3 \\ \mathcal{F}(V) &= \mathbb{Z}/3 \\ \mathcal{F}(\emptyset) &= \{e\} \end{aligned}$$

   with restriction maps $\mathrm{res}^X_U(a,b) = a \bmod 3$ and $\mathrm{res}^X_V(a,b) = b \bmod 3$.

   Then $\mathcal{F}$ is not a sheaf. Why?

4. *Skyscraper sheaves* Let X be a topological space. Let $P \in X$ be a point whose closure is Z.

   (a) Suppose $Q \in Z$. Let U be an open neighborhood of Q. Show that $P \in U$.

   (b) Let A be a finite abelian group. Define a sheaf $\mathcal{S}$ as follows:

   $$\mathcal{S}(U) = \begin{cases} A & P \in U \\ \{0\} & P \notin U \end{cases}$$

   (with the obvious restriction maps). Compute the stalks of $\mathcal{S}$: show that

   $$\mathcal{S}_Q = \begin{cases} A & Q \in Z \\ \{0\} & \text{otherwise} \end{cases}.$$

   Also, compare with [GW]2.14.

5. [GW]2.10.