Homework 1
Due: Friday, August 26

1. (a) Let $Y \subset \mathbb{A}^n$ be any subset. Prove that

$$\mathcal{I}(Y) := \{f \in k[x_1, \cdots, x_n] : \forall P \in Y, f(P) = 0\}$$

is an ideal of $k[x_1, \cdots, x_n]$.

(b) Given an example of an n and Y such that $\mathcal{Z}(\mathcal{I}(Y)) \neq Y$.

2. (a) Define affine sets by

$$X = \left\{(a_1, a_2) : a_2^2 = a_1^3 - a_1\right\} \subset \mathbb{A}^2$$
$$Y = \left\{(b_1, b_2, b_3) : b_2^2 + b_1 = b_3 b_1 \text{ and } b_1^2 = b_3\right\} \subset \mathbb{A}^3$$

Consider the map

$$\mathbb{A}^2 \xrightarrow{\alpha} \mathbb{A}^3$$

$$(a_1, a_2) \longmapsto (a_1, a_2, a_1^2)$$

Show that $\alpha(X) \subseteq Y$.

(b) Define ideals by

$$I = (x_2^2 - x_1^3 + x_1) \subset k[x_1, x_2]$$
$$J = (t_2^2 + t_1 - t_3 t_1, t_1^2 - t_3) \subset k[t_1, t_2, t_3]$$

Consider the map

$$k[t_1, t_2, t_3] \xrightarrow{\beta} k[x_1, x_2]$$

$$t_1 \longmapsto x_1$$

$$t_2 \longmapsto x_2$$

$$t_3 \longmapsto x_1^2$$

Show that $\beta(J) \subseteq I$.