Homework 4

Due: Friday, October 7

1. Early in the course, we made the following provisional definition:

   Let us call a family of (holomorphic) functions $\mathcal{F}$ on a connected set Ω normal if, for every compact set $K \subset \Omega$ and every sequence $\{f_n\}$ of elements of $\mathcal{F}$, either:

   - There exists a subsequence which is uniformly Cauchy. Specifically, there is a subsequence $\{f_{n_j}\}$ such that for every $\epsilon > 0$ there exists $N = N_\epsilon$ such that if $j, k > N$ then $\left\|f_{n_j} - f_{n_k}\right\|_K < \epsilon$; or
   - There is a uniformly divergent subsequence. Specifically, there is a subsequence $\{f_{n_j}\}$ such that f_{n_j} is nonvanishing on K; and for every $\epsilon > 0$ there exists $N = N_\epsilon$ such that if $j > N$, then $\left\|1/f_{n_j}\right\|_K < \epsilon$.

   Prove or disprove:

   > A set of holomorphic functions on Ω is normal in this sense if and only if it's normal, as a subset of $\mathrm{Hol}(\Omega, \mathbb{C})$, in the sense of [M, Chapter 3].

2. [M] 3-b

3. [M] 3-c

4. [M] 3-e

Read [M] 3-f.