Homework 6

Due: Friday, September 28

1. Consider a trivial theta function $\theta(z) = \exp(pz^2 + qz + r)$. Describe its type $\{(a_\lambda, b_\lambda) : \lambda \in \Lambda\}$.

2. Let $\Lambda \subset \mathbb{C}$ be a lattice. Suppose there is a theta function of type $\{(a_\lambda, b_\lambda) : \lambda \in \mathbb{C}\}$.

   (a) Show that for all $\lambda, \mu \in \Lambda$,
   $$a_{\lambda+\mu} = a_\lambda + a_\mu.$$

   (b) What condition must the b_λ satisfy?

   (HINT: *Let θ be of type $\{a_\lambda, b_\lambda\}$. Evaluate $\theta(z + \lambda + \mu)$ in two different ways.*)

3. Fix a lattice Λ. Suppose α is a theta function of type $\{a_\lambda, b_\lambda\}$, and β is a theta function of type $\{c_\lambda, d_\lambda\}$. Show that the product $\alpha\beta$ is also a theta function. What is its type?

4. Let θ be a theta function (for the lattice Λ, with type $\{(a_\lambda, b_\lambda)\}$). Show that $(\theta'/\theta)'$ is periodic with respect to Λ.

5. Let $\xi = \frac{1+\tau}{2}$. In class on Monday[1] we will define functions

$$\begin{aligned}
\theta_{00}(z) &= \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (z) = \vartheta(z) \\
\theta_{01}(z) &= \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} (z) = \vartheta(z + \frac{1}{2}) \\
\theta_{10}(z) &= \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} (z) = \vartheta(z + \frac{\tau}{2})\widetilde{e}(\tau/8 + \frac{1}{2}z) \\
\theta_{11}(z) &= \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} (z) = \vartheta(z + \frac{\tau+1}{2})\widetilde{e}(\tau/8 + \frac{1}{2}(z + \frac{1}{2}))
\end{aligned}$$

   and use identities

$$\begin{aligned}
\theta_{00}(z + \xi) &= -i\widetilde{e}(-\tau/8 - z/2)\theta_{11}(z) \\
\theta_{01}(z + \xi) &= \widetilde{e}(-\tau/8 - z/2)\theta_{10}(z) \\
\theta_{10}(z + \xi) &= -i\widetilde{e}(-\tau/8 - z/2)\theta_{01}(z) \\
\theta_{11}(z + \xi) &= -\widetilde{e}(-\tau/8 - z/2)\theta_{00}(z).
\end{aligned}$$

   Verify these identities.

[1] actually, Wednesday