1. Given elements $\alpha, \beta \in \mathbb{C}$, the (additive) subgroup they generate is:

$$G_{\alpha,\beta} = \{ m\alpha + n\beta : m, n \in \mathbb{Z} \}$$

(a) Give an example where $G_{\alpha,\beta} \cong \mathbb{Z}$.

(b) Give an example where $G_{\alpha,\beta} \cong \mathbb{Z} \oplus \mathbb{Z}$, but $G_{\alpha,\beta}$ is not a lattice.

2. Suppose $\Lambda \subset \mathbb{C}$ is a lattice, and $f$ is a holomorphic function which is $\Lambda$-periodic. Prove that $f$ is constant.

3. Suppose $\Lambda \subset \mathbb{C}$ is a lattice, with ordered basis $\{ \omega_1, \omega_2 \}$. Let $D$ be the (open) parallelogram $\{ s\omega_1 + t\omega_2 : 0 < s, t < 1 \}$, and let $C$ be a simple, positive closed contour around the boundary of $\overline{D}$. (For example, $C$ is the piecewise-linear contour which visits, successively, 0, $\omega_1$, $\omega_1 + \omega_2$, $\omega_2$, and back to 0.)

Suppose that $f$ is meromorphic, $\Lambda$-periodic, and has no poles along $C$. Show that $\int_C f(z) \, dz = 0$. 