Homework 1

Due: Friday, August 24

1. Given elements $\alpha, \beta \in \mathbb{C}$, the (additive) subgroup they generate is:

$$G_{\alpha,\beta} = \{m\alpha + n\beta : m, n \in \mathbb{Z}\}$$

   (a) Give an example where $G_{\alpha,\beta} \cong \mathbb{Z}$.

   (b) Give an example where $G_{\alpha,\beta} \cong \mathbb{Z} \oplus \mathbb{Z}$, but $G_{\alpha,\beta}$ is *not* a lattice.

2. Suppose $\Lambda \subset \mathbb{C}$ is a lattice, and f is a holomorphic function which is Λ-periodic. Prove that f is constant.

3. Suppose $\Lambda \subset \mathbb{C}$ is a lattice, with ordered basis $\{\omega_1, \omega_2\}$. Let D be the (open) parallelogram $\{s\omega_1 + t\omega_2 : 0 < s, t < 1\}$, and let C be a simple, positive closed contour around the boundary of $\overline{D}$. (For example, C is the piecewise-linear contour which visits, successively, 0, ω_1, $\omega_1 + \omega_2$, ω_2, and back to 0.)

   Suppose that f is meromorphic, Λ-periodic, and has no poles along C. Show that $\int_C f(z)dz = 0$.

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