Homework 1
Due: Friday, January 30

1. Let C be the ellipse
$$2x^2 + y^2 = 1.$$
Prove that $C(\mathbb{Q})$ is infinite.

2. (a) Use descent to show that
$$x^2 = 2y^2$$
has no integer solution. (HINT: *x is even.*)
(b) Use descent to show that
$$x^3 + 2y^3 + 4z^3 = 0$$
has no integer solution other than $(0, 0, 0)$. (HINT: *x is even.*)

3. (a) Consider the Bachet curve
$$E : y^2 = x^3 - 2$$
and the point
$$P = (3, 5) \in E(\mathbb{Q}).$$
Let L_m be the line through P of slope m. Show, by example, that
$$L_m \cap E$$
need not consist of rational points.
(b) In this respect, why does E behave so differently than C (from problem (1))?