Homework 6

Due: Wednesday, March 4

1. Consider $V = \mathbb{R}[x]_3$, with basis $\mathcal{B} = \{1, x, x^2, x^3\}$.
   (a) Describe the dual basis $\mathcal{B}^*$.
   (b) Consider $I \in V^*$ defined by $I(p(x)) = \int_{-2}^{3} p(x)dx$. Express I as a linear combination of elements of the dual basis from part (a).
2. Suppose $A \subset V$; its annihilator is

   $$A^\perp = \{v^* \in V^* : \forall v \in A, (v, v^*) = 0\}.$$

   Show that $A^\perp$ is a subspace of V^*.
3. [KK]3.2.3. (HINT: *Consider* (Tv, v^*).)
4. Suppose $T \in \mathcal{L}(W, V)$ is injective. Show that the adjoint map $T^* \in \mathcal{L}(V^*, W^*)$ is surjective.