Homework 3

Due: Wednesday, February 11

1. Let $V = \mathbb{R}^3$, and let W be the subspace

$$W = \{ \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix} : t \in \mathbb{R} \}.$$

   (a) Suppose $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ are two elements of V. How can you tell if $x \equiv y \bmod W$? Explain.

   (b) Describe a function $T : V \to \mathbb{R}^2$ such that $T(x) = T(y)$ if and only if $x \equiv y \bmod W$.

2. Let V be a vector space over $\mathbb{F}$, and let $W \subset V$ be a subspace. As in class, if $v \in V$ let $\widetilde{v} = v + W$; it's the set of all $z \in V$ such that $v \equiv z \bmod W$.

   Suppose $v_1, v_2 \in V$ satisfy $\widetilde{v}_1 = \widetilde{v}_2$, and suppose $a \in \mathbb{F}$. Show that $\widetilde{av_1} = \widetilde{av_2}$. (HINT: *See [KK], Section 1.2.4.*)

3. [KK] 2.1.2.

4. (a) Consider the function

$$\begin{array}{ccc} \mathbb{Q}^3 & \xrightarrow{S} & \mathbb{Q}^2 \\ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} & \longmapsto & \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix} \end{array}$$

   Show that S is a linear transformation.

   (b) Conisder the function

$$\begin{array}{ccc} \mathcal{C}^\infty(-\infty, \infty) & \xrightarrow{T} & \mathbb{R}[x] \\ f & \longmapsto & f(0) + f'(0)x + \frac{f''(0)}{2}x^2 \end{array}$$

   Show that T is a linear transformation.

5. The complex conjugation map from $\mathbb{C}$ to $\mathbb{C}$ is:

$$\begin{aligned} \mathbb{C} &\xrightarrow{B} \mathbb{C} \\ z &\longmapsto \overline{z} \\ a + bi &\longmapsto a - bi \end{aligned}$$

(a) Think of $\mathbb{C}$ as a vector space over $\mathbb{R}$. Is B a linear transformation? Explain.

(b) Think of $\mathbb{C}$ as a vector space over $\mathbb{C}$. Is B a linear transformation? Explain.