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Homework 1  
Due: Wednesday, January 28

*In this and all subsequent assignments, [KK] refers to A (terse) introduction to linear algebra, Y. Katznelson and Y. Katznelson, American Mathematical Society, 2008. More precisely, [KK] 1.1.3 means the problem labeled ex1.1.3 in Section 1.1 of Katznelson and Katznelson.*

1. [KK]1.1.3.
2. [KK]1.2.1.
3. Let  $V$  be a vector space, and let  $U$  and  $W$  be subspaces.
  - (a) Prove that the intersection  $U \cap W$  is also a subspace of  $V$ .
  - (b) Given an example showing that  $U \cup W$  need not be a subspace of  $V$ .

*See also [KK] 1.2.3-1.2.4.*

4. Consider the set

$$S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x^2 - y^2 = 0 \right\} \subset \mathbb{R}^2$$

- (a) Is  $S$  closed under scalar multiplication? Prove or give a counterexample.
  - (b) Is  $S$  closed under addition? Prove or give a counterexample.
5. Consider the set

$$P = \{f(x) \in \mathcal{C}^\infty(-\infty, \infty) : \forall a \in \mathbb{R}, f(a) = f(a + 2\pi)\}.$$

Show that  $P$  is a subspace of  $\mathcal{C}^\infty(-\infty, \infty)$ .