Homework 8  
Due: Friday, October 19

1. If $f$ is a function, and $S = z_1, \ldots, z_n$ is a finite set of complex numbers, then the average value of $f$ on $S$ is

$$\langle f(z) \rangle_S = \frac{1}{n} \sum_{j=1}^{n} f(z_j).$$

Fix a number $n \geq 2$ and a nonzero number $\alpha$. Let $S$ be the set of $n^{th}$ roots of $\alpha$.

(a) What is $\langle z \rangle_S$? (HINT: See problem 2b on the midterm.)
(b) Suppose $1 \leq m < n$. What is $\langle z^m \rangle_S$?
(c) Suppose $m = 0$. What is $\langle z^m \rangle_S$?
(d) Let $P(z)$ be a polynomial of degree $\deg P < n$. Prove that

$$\langle P(z) \rangle_S = P(0).$$


4. (a) [BC] 40.2.
   (b) Repeat problem (a) using the function $g(z) = \overline{z} - 1.$