Homework 8

Due: Friday, October 19

1. If f is a function, and $S = z_1, \cdots, z_n$ is a finite set of complex numbers, then the average value of f on S is

$$\langle f(z) \rangle_S = \frac{1}{n} \sum_{j=1}^{n} f(z_j).$$

Fix a number $n \geq 2$ and a nonzero number α. Let S be the set of n^{th} roots of α.

   (a) What is $\langle z \rangle_S$? (HINT: *See problem 2b on the midterm.*)

   (b) Suppose $1 \leq m < n$. What is $\langle z^m \rangle_S$?

   (c) Suppose $m = 0$. What is $\langle z^m \rangle_S$?

   (d) Let $P(z)$ be a polynomial of degree $\deg P < n$. Prove that

$$\langle P(z) \rangle_S = P(0).$$

2. [BC]37.1.

3. [BC]37.3, 40.7.

4. (a) [BC] 40.2.

   (b) Repeat problem (a) using the function $g(z) = \overline{z} - 1$.