Homework 7
Due: Friday, October 5

Remember, there's a midterm on Monday, October 8. Start studying!

1. [BC] 30.2, 30.5.

2. [BC] 30.10.

3. For each of the expressions α^β in [BC] 32.2:
   - Find all values of α^β.
   - Find the principal value of α^β.

4. If $f(z) = u(x, y) + iv(x, y)$ is differentiable at z, then
$$f'(z) = u_x + iv_x. \tag{1}$$
   (a) Using the relations
$$\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta), \end{aligned}$$
   show that
$$\begin{aligned} u_x &= u_r\cos(\theta) - u_\theta\frac{\sin(\theta)}{r} \\ u_y &= u_r\sin(\theta) + u_\theta\frac{\cos(\theta)}{r}. \end{aligned}$$
   (b) Show that the following alternate form of (1) holds: If $z = r\exp(i\theta)$, then
$$f'(z) = \exp(-i\theta)(u_r + iv_r).$$
   (HINT: *See [BC]22.7-8 and p.66*)

5. Use the result of the previous problem to show that if $L(z)$ is a branch of the logarithm, then $L'(z) = \frac{1}{z}$.