Homework 5

Due: Friday, September 21

1. [BC] 17.1.

2. [BC] 17.5.

3. (a) [BC] 19.2.a

   (b) Fix a natural number n. Show, by induction on j, that the j^{th} derivative of z^n is

$$\frac{d^j}{dz^j} z^n = \begin{cases} \frac{n!}{(n-j)!} z^{n-j} & 1 \le j < n \\ n! & j = n \\ 0 & j > n \end{cases} .$$

   (You may use the fact that if $n \ge 1$, then $\frac{d}{dz} z^n = n z^{n-1}$.)

   (c) Show that the j^{th} derivative of P, evaluated at 0, is

$$P^{(j)}(0) = j! a_j.$$

4. Consider the function $f(z) = \overline{z}$.

   (a) Prove that f is continuous everywhere.

   (b) Prove that f is not differentiable anywhere.

5. Let $P(z) = (z - z_1) \cdots (z - z_n)$. Prove, by induction on the degree n, that

$$\frac{P'(z)}{P(z)} = \frac{1}{z - z_1} + \frac{1}{z - z_2} + \cdots + \frac{1}{z - z_n}.$$