## Homework 3

Due: Friday, September 7

1. [BC]9.1, 9.2.

2. [BC]9.5.

3. Suppose that $\zeta$ is an $n^{th}$ root of unity, but $\zeta \neq 1$.

   (a) Prove that
   $$1 + \zeta + \zeta^2 + \zeta^3 + \cdots + \zeta^{n-1} = 0.$$

   (b) Let $m$ be a natural number, and suppose that $\gcd(m, n) = 1$. Show that
   $$\zeta^m \neq 1.$$
   (HINT: *See HW 1#2.*)

   (c) Let $m$ be a natural number, and suppose that $\gcd(m, n) = 1$. Show that
   $$1 + \zeta^m + \zeta^{2m} + \zeta^{3m} + \cdots + \zeta^{(n-1)m} = 0.$$

4. Give two different proofs of the following statement:

   If $|z| = 1$, then $\overline{z} = z^{-1}$.

5. This problem concerns the equation
   $$(z + 1)^{10} = (z - 1)^{10}. \tag{1}$$

   (a) Show directly that if $z$ is a solution to Equation (1), then $z$ must also satisfy
   $$|z + 1| = |z - 1|. \tag{2}$$
   What geometric object does Equation (2) describe?
   *Note: We are* not *saying that any solution to* (2) *also satisfies* (1)*!*

   (b) Divide both sides of Equation (1) by $(z - 1)^{10}$, so that the equation is of the form $w^{10} = 1$. Use this to find all solutions $z$ to Equation (1).

   (c) For each solution $z$ to Equation (1), compute $\overline{z}$.
   (HINT: *Use the result of problem 4.*)