Homework 12

Due: Friday, November 16

1. [BC]52.6, 52.8.
2. Carefully prove the following assertion from class:

   Consider a series $\sum_{n\geq 1} z_n$, with partial sums S_N, and let S be a complex number. Define the N^{th} remainder as $\rho_N = S - S_N$.

   Show that $\sum_{n\geq 1} z_n = S$ if and only if the sequence $\{\rho_N\}_{N\geq 1}$ has limit 0.

3. [BC]54.7.
4. *Without* computing derivatives, find the Maclaurin series for $\cos^3(z)$. (HINT: *Use the fact that* $4\cos^3(z) = \cos(3z) + 3\cos(z)$.)
5. Suppose a function $f(z)$ is represented in a neighborhood of 0 by the power series

$$f(z) = \sum_{n\geq 0} a_n z^n.$$

   (a) Write down the Maclaurin series for $f'(z)$.

   (b) Prove that if $f(z) = f'(z)$ then $a_{n+1} = \frac{1}{n+1} a_n$.

   (c) Let λ be a nonzero complex number. Suppose that $f'(z) = \lambda f(z)$. Give a formula for a_n in terms of a_0.