Homework 1
Due: Friday, August 24

In this and all subsequent assignments, [BC] *refers to* Complex Variables and Applications*, J. Brown and R. Churchill, seventh edition, McGraw-Hill, 2004. More precisely,* [BC] 25.3 *means problem 3 from Section 25 of Brown and Churchill.*

1. [BC]2.2, 2.4, 3.1.
2. Prove that if $z \neq 1$ and if n is a natural number, then
$$1 + z + z^2 + \cdots + z^n = \frac{z^{n+1} - 1}{z - 1}.$$
3. Here is one way of deriving Cardano's solution to the depressed cubic
$$x^3 = 3px + 2q. \tag{1}$$
(a) Show that if you can find s and t simultaneously satisfying
$$st = p \tag{2}$$
and
$$s^3 + t^3 = 2q, \tag{3}$$
then $x = s + t$ is a solution to (1).
(This reduces the study of Equation (1) to that of Equations (2) and (3).)
(b) Use Equation 2 to eliminate t from Equation 3. Work to obtain a polynomial of degree 6 in s. Note that it is also a degree 2 polynomial in s^3.
(c) Use the quadratic formula to obtain the two possible values of s^3. For a given value of s^3, what must t^3 be?
(d) Deduce Cardano's solution:
$$x = \sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}}.$$