Homework 8

Due: Friday, October 20

1. (a) [BC] 40.2.

   (b) Repeat problem (a) using the function $g(z) = \overline{z} - 1$.

2. [BC] 37.3, 40.7.

3. [BC] 40.10.

4. Let C be the contour which traces the circle $|z| = 2$ once in the counterclockwise direction. Find an upper bound for

$$\left| \int_C \frac{e^z}{z^2+1} dz \right|.$$

(HINT: *Do not try to explicitly evaluate this integral.*)

5. (a) Let $P(z)$ be a polynomial, and let C be any closed contour. Prove that $\int_C P(z)dz = 0$.

   (b) Let f be a function defined on a domain D, and let $C \subset D$ be a closed contour. Suppose you know that for every $\epsilon > 0$ there exists some polynomial $P_\epsilon(z)$ such that, for every point z on the contour C, $|f(z) - P_\epsilon(z)| < \epsilon$.
   Prove that $\int_C f(z)dz = 0$.