The first problem addresses the extra credit on the midterm:

1. If \( f \) is a function, and \( S = z_1, \ldots, z_n \) is a finite set of complex numbers, then the average value of \( f \) on \( S \) is

\[
\langle f(z) \rangle_S = \frac{1}{n} \sum_{j=1}^{n} f(z_j).
\]

Fix a number \( n \geq 2 \) and a nonzero number \( \alpha \). Let \( S \) be the set of \( n^{th} \) roots of \( \alpha \).

(a) Suppose \( 1 \leq m < n \). What is \( \langle z^m \rangle_S \)?
(b) Suppose \( m = 0 \). What is \( \langle z^m \rangle_S \)?
(c) Let \( P(z) \) be a polynomial of degree \( \deg P < n \). Prove that

\[
\langle P(z) \rangle_S = P(0).
\]

2. [BC] 37.1

3. [BC] 37.4