Homework 9

Due: Wednesday, April 18

1. Prove that the system of functions $\{\cos(mx), \sin(nx)\}_{n\in\mathbb{Z}}$ is orthogonal with respect to the inner product $\langle f, g\rangle = \int_0^{2\pi} f(x)\overline{g(x)}\,dx$. Explicitly, show that

$$\langle \cos(mx), \cos(nx)\rangle = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$$
$$\langle \sin(mx), \sin(nx)\rangle = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases}$$
$$\langle \sin(mx), \cos(nx)\rangle = 0$$

2. Denote the n^{th} Fourier coefficient of f by $c_n(f)$.

   (a) Show that $c_n(f+g) = c_n(f) + c_n(g)$.

   (b) Suppose $a \in \mathbb{R}$. Show that $c_n(af) = ac_n(f)$.

   (HINT: *These are easy if you're willing to use the inner-product formalism.*)

3. [F]8.1.3

4. [F]8.1.4

5. [F]8.2.4ab