Homework 3

Due: Wednesday, Feburary 8

1. *Orientation* A Möbius strip S in $\mathbb{R}^3$ can be parametrized as

$$W \xrightarrow{\vec{G}} \mathbb{R}^3\{(u,v) : u \in [0,2\pi], v \in [-1,1]\}$$

$$(u,v) \longmapsto (G_1(u,v), G_2(u,v), G_3(u,v))$$

where

$$\begin{aligned} G_1(u,v) &= (1 + \frac{1}{2}v\cos(\frac{1}{2}u))\cos(u) \\ G_2(u,v) &= (1 + \frac{1}{2}v\cos(\frac{1}{2}u))\sin(u) \\ G_3(u,v) &= \frac{1}{2}v\sin(\frac{1}{2}u) \end{aligned}$$

   (a) Verify that $\vec{G}(0,0) = \vec{G}(2\pi,0)$.
   Now attempt to assign an orientation using $\tau_1 = \frac{\partial}{\partial u}\vec{G}$, $\tau_2 = \frac{\partial}{\partial v}\vec{G}$.

   (b) Use this to compute the normal vector at $\vec{G}(0,0)$.

   (c) Use this to compute the normal vector at $\vec{G}(2\pi,0)$.

2. [F] 5.5.2

3. [F] 5.5.3

4. [F] 5.8.1

5. [F] 5.8.3