## Homework 2

Due: Wednesday, February 2

1. [F] 5.1.5.
2. [F] 5.1.6.
3. Prove Green's theorem in the special case where the domain $S$ is a rectangle.

In somewhat more detail, consider the rectangle $S$ with boundary $\partial S=\cup_{1 \leq i \leq 4} C_{i}$, as follows:


Let $\vec{F}(x, y)=\left(F_{1}(x, y), F_{2}(x, y)\right)$. Remember, $\vec{F} d \vec{x}=F_{1}(x, y) d x+F_{2}(x, y) d y$.
(a) Write down parametrizations for each curve $C_{i}$.
(b) For each $C_{i}$, compute $\int_{C_{i}} F_{1}(x, y) d x$ and $\int_{C_{i}} F_{2}(x, y) d y$.
(c) Compute $\iint_{S} \frac{\partial F_{2}}{\partial x} d x d y$ and $\iint_{S} \frac{\partial F_{1}}{\partial y} d x d y$.
(d) Deduce Green's theorem:

$$
\int_{\partial S} \vec{F} d \vec{x}=\iint_{S}\left(\frac{\partial F_{2}}{\partial x}-\frac{\partial F_{1}}{\partial y}\right) d x d y .
$$

4. [F] 5.2.1.
