## Homework 2 Due: Wednesday, February 2

1. [F] 5.1.5.

- 2. [F] 5.1.6.
- 3. Prove Green's theorem in the special case where the domain *S* is a rectangle. In somewhat more detail, consider the rectangle *S* with boundary  $\partial S = \bigcup_{1 \le i \le 4} C_i$ , as follows:



Let  $\vec{F}(x,y) = (F_1(x,y), F_2(x,y))$ . Remember,  $\vec{F}d\vec{x} = F_1(x,y)dx + F_2(x,y)dy$ .

- (a) Write down parametrizations for each curve  $C_i$ .
- (b) For each  $C_i$ , compute  $\int_{C_i} F_1(x, y) dx$  and  $\int_{C_i} F_2(x, y) dy$ .
- (c) Compute  $\iint_{S} \frac{\partial F_2}{\partial x} dx dy$  and  $\iint_{S} \frac{\partial F_1}{\partial y} dx dy$ .
- (d) Deduce Green's theorem:

$$\int_{\partial S} \vec{F} d\vec{x} = \iint_{S} \left( \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx dy.$$

4. [F] 5.2.1.

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