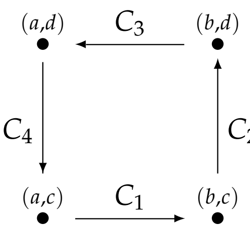
Homework 2

Due: Friday, Feburary 5

1. Prove Green's theorem in the special case where the domain S is a rectangle.

   In somewhat more detail, consider the rectangle S with boundary $\partial S = \cup_{1 \le i \le 4} C_i$, as follows:

   Let $\vec{F}(x,y) = (F_1(x,y), F_2(x,y))$. *Remember,* $\vec{F}d\vec{x} = F_1(x,y)dx + F_2(x,y)dy$.

   (a) Write down parametrizations for each curve C_i.

   (b) For each C_i, compute $\int_{C_i} F_1(x,y)dx$ and $\int_{C_i} F_2(x,y)dy$.

   (c) Compute $\iint_S \frac{\partial F_2}{\partial x} dxdy$ and $\iint_S \frac{\partial F_1}{\partial y} dxdy$.

   (d) Deduce Green's theorem:

   $$\int_{\partial S} \vec{F}d\vec{x} = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dxdy.$$

2. Let S be a regular region in $\mathbb{R}^2$. Show that its area is $\int_{\partial S} xdy$. (HINT: *By definition, the area of S is* $\iint_S 1 \cdot dxdy$.)

3. [F] 5.2.1. (In other words, do problem #1 in section 5.2.)

4. [F] 5.3.3. (HINT: *The book's parametrization gives a map* $\vec{G} : \mathbb{R}^2 \to \mathbb{R}^3$, *where the coordinates on* $\mathbb{R}^2$ *are* ϕ *and* θ.)

5. [F] 5.4.8.