Homework 10

Due: Friday, April 16

1. [F] 7.3.10.
2. Prove the trigonometric identities

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\sin(a+b) = \cos(a)\sin(b) + \sin(a)\cos(b)$$

   in two different ways:

   (a) Using the characterization $\exp(ix) = \cos(x) + i\sin(x)$; and

   (b) Using the uniqueness (from class, Monday April 12) of solutions to the initial value problem

$$\begin{aligned} f'(x) &= -g(x) \\ g'(x) &= f(x) \\ f(0) &= a \\ g(0) &= b \end{aligned}$$

3. [F]7.4.1.
4. Let f be a differentiable function such that $f(x+y) = f(x)f(y)$ for all real x and y. Show that

$$f'(x) = f'(0)f(x).$$

   (HINT: *Calculate $f'(x)$ using the definition of the derivative.*)

5. Find power series expansions, valid on $(-1,1)$, for the following functions:

   (a) $f(x) = \frac{1}{1-x^2}$;

   (b) $g(x) = \frac{1}{(1-x)^2}$;

   (c) $h(x) = \frac{1}{(1-x^2)^2}$.

   (HINT: *As a warmup for (b), calculate $(\sum_{n=0}^{5} x^j)^2$. If $N \geq j$, what is the coefficient of x^j in $(\sum_{n=1}^{N} x)^2$?*)