Homework 4 Due: Friday, September 16

1. (a) Show that the identity function

$$\mathbb{R}^n \xrightarrow{\text{id}} \mathbb{R}^n$$
$$\vec{x} \longmapsto \vec{x}$$

is continuous.

(b) Fix an element $\vec{b} \in \mathbb{R}^m$. Show that the constant function

$$\mathbb{R}^n \xrightarrow{\kappa} \mathbb{R}^m$$
$$\vec{x} \longmapsto \vec{b}$$

is continuous.

- 2. Let $\{a_j\}_{J=0}^{\infty}$ be a sequence of elements in \mathbb{R}^n , and suppose $\vec{v} \in \mathbb{R}^n$.
 - (a) Suppose that $\lim_{j\to\infty} \vec{a}_j = \vec{v}$. Show that for each $\epsilon > 0$, the set

$$\{j: \vec{a}_j \notin B_{\epsilon}(\vec{v})\} \tag{1}$$

is finite.

(b) Show the converse of (a). In other words, suppose that for each $\epsilon > 0$, the set defined in (1) is finite. Show that

$$\lim_{j\to\infty}\vec{a}_j=\vec{v}.$$

- 3. [F] 1.4.2
- 4. [F] 1.4.4
- 5. [F]1.4.6

Professor Jeff Achter Colorado State University