Gauss-Jordan elimination

Given an $m \times n$ matrix A, augmented or not, our goal is to transform it into one which is in reduced row echelon form.

First, put A in echelon form. Essentially, one steps down the rows, making exchanges so that the first nonzero entry in a given row happens before the first nonzero entry in any lower row, and then subtracting off as appropriate to get the staircase appearance:

```
for cur_row from 1 to m do
: make sure pivot column in current row is as small as possible
  for higher_row from cur_row+1 to m do
    if (first_nonzero_entry(cur_row) > first_nonzero_entry(higher_row)) then
      exchange rows cur_row and higher_row
: make pivot entry be 1
  pivot_col = first_nonzero_entry(cur_row)
  rescale(cur_row, 1/A[cur_row,pivot_col])
  for higher_row from cur_row+1 to m do
     add (-1)*A[higher_row, pivot_col]*cur_row to higher_row
```

At this point, we have a matrix in echelon form. Now work back up the chain, subtracting off multiples of *lower* rows, in order to eliminate spurious entries in each pivot column:

```
for cur_row from m down to 2 do
  pivot_col = first_nonzero_entry(cur_row)
  for lower_row from m-1 down to 1 do
      add (-1)*A[lower_row,pivot_col]*cur_row to lower_row
```

Try writing down a matrix and see how the algorithm unfolds.