Homework 8

Due: Friday, April 14

Recall that if V is an inner product space with a basis $\mathcal{B} = \{v_1, \cdots, v_n\}$, then the Gram matrix of the inner product with respect to $\mathcal{B}$ is the matrix whose ij^{th} entry is $\langle v_i, v_j \rangle$.

1. Consider $\mathbb{R}^3$ with the standard basis and the standard inner product. Find an orthonormal basis for $U = \operatorname{span}((1,2,2)^T, (1,3,1)^T)$.

2. On $\mathcal{P}_2(\mathbb{R})[x]$, consider the inner product given by

$$\langle p, q \rangle = \int_0^1 p(x)q(x)dx.$$

   Apply the Gram-Schmidt procedure to the basis $\{1, x, x^2\}$ to produce an orthonormal basis of $\mathcal{P}_2(\mathbb{R})[x]$

3. Let $V = \mathbb{R}^3$ with an inner product whose Gram matrix, with respec to the standard basis $\mathcal{B}$, is

$$G_{\mathcal{E}} = \begin{pmatrix} 2 & 2 & -6 \\ 2 & 3 & -5 \\ -6 & -5 & 22 \end{pmatrix}$$

   (a) Calculate the inner product of $(1,2,3)^T$ with $(4,5,6)^T$.

   (b) Let $U = \operatorname{span}((1,0,0)^T, (0,1,0)^T)$. Calculate a basis of $U^\perp$.

4. Suppose that U is a subspace of an inner product space V. Carefully prove that $U^\perp$ is also a subspace of V.

5. Suppose that U is a subspace of an inner product space V. Prove that

$$\dim U^\perp = \dim V - \dim U.$$