Homework 5

Due: Friday, March 10

1. Suppose that W and V are finite-dimensional vector spaces over $\mathbb{F}$, with bases $\mathcal{C}$ and $\mathcal{B}$, respectively.

   (a) Suppose that $v \in V$ and $\lambda \in \mathbb{F}$. Show that $[\lambda v]_\mathcal{B} = \lambda [v]_\mathcal{B}$.

   (b) Suppose that $f \in \mathcal{L}(W, V)$; then the function $\lambda f \in \mathcal{L}(W, V)$. Show that $[\lambda f]_{\mathcal{B}\leftarrow\mathcal{C}} = \lambda [f]_{\mathcal{B}\leftarrow\mathcal{C}}$.

   (Don't be scared of all the definitions and symbols; this isn't too hard when you unwind it. This fills in the missing part of the proof from class that the function $\mathcal{L}(W, V) \to \mathrm{Mat}(m, n, \mathbb{F})$ given by $f \mapsto [f]_{\mathcal{B}\leftarrow\mathcal{C}}$ is a linear transformation.)

2. Let $V = \mathbb{R}^2$ with the standard basis $\mathcal{E} = \{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\}$. Let $f : V \to V$ be the linear transformation with matrix

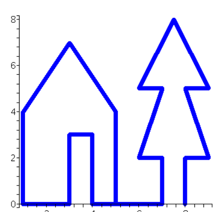
   $$[f]_{\mathcal{E}\leftarrow\mathcal{E}} = \begin{pmatrix} -3 & -8 \\ 10 & 27 \end{pmatrix}.$$

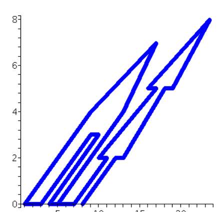
   (a) Suppose $[v_1]_\mathcal{E} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$. What is $[f(v_1)]_\mathcal{E}$?

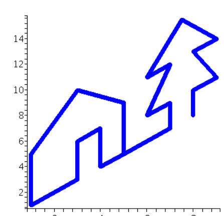
   (b) Let $\mathcal{C}$ be the basis $\mathcal{C} = \{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix}\}$. What is $[f]_{\mathcal{C}\leftarrow\mathcal{C}}$?

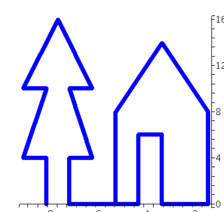
   (c) Suppose $[v_2]_\mathcal{C} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. What is $[f(v_2)]_\mathcal{C}$?

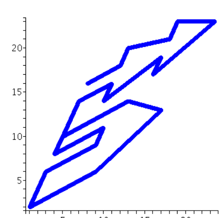
3. Let $V = \mathbb{R}^2$ with the standard basis $\mathcal{E} = \{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\}$, and consider the following pictures:

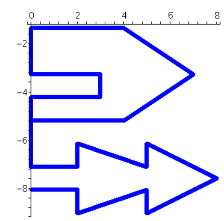
(1)


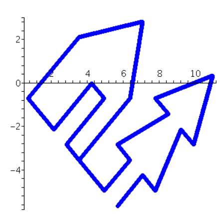
(2)


(3)


(4)


(5)


(6)


(7)


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M369 Linear Algebra
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(a) A linear transformation $f_{21} : V \to V$ transforms picture (1) to picture (2). What is $[f_{21}]_{\mathcal{E}\leftarrow\mathcal{E}}$?

(b) Do the same thing for the rest of the pictures; for each $j = 2, \cdots, 7$, find the matrix of the linear transformation f_{j1} which transforms picture (1) to picture (j).

4. *(Continuation of problem 3)*

   (a) Find a matrix for the transformation of (2) into (5). (Again, use the standard basis $\mathcal{E}$.)

   (b) For each matrix you find in problem 3, compute the determinant of the matrix of the transform. How does the sign of the determinant relate to the picture?

5. Let U_1 and U_2 be subspaces of a vector space V. Suppose that $f \in \mathcal{L}(V, V)$, and that each of U_1 and U_2 is invariant under f. Prove that $U_1 \cap U_2$ is invariant under f.