Homework 2

Due: Friday, February 10

Remark: We're now writing vectors as column vectors, but it's still easier to typeset things as row vectors. Do get around this, I will sometimes describe the transpose of a vector, rather than the vector itself. Thus,

$$(1,2,3)^T \text{ means } \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Warmup: For what value(s) of h is $(1,-2,h)^T$ in $\text{span}((3,4,5)^T, (-1,-3,2)^T)$? (You need not hand this in.)

1. Which of the following sets are spanning sets for $\mathbb{R}^3$? In each case, if the set spans $\mathbb{R}^3$, prove it; if not, write down a vector which is *not* in the span of the set in question.

   (a) $\{(1,1,1)^T\}$

   (b) $\{(1,2,0)^T, (0,1,0)^T, (0,2,1)^T\}$

   (c) $\{(1,2,0)^T, (0,1,0)^T, (0,2,1)^T, (1,2,3)^T\}$

   (d) $\{(1,2,-2)^T, (2,3,-2)^T, (2,2,0)^T, (3,4,-2)^T\}$

   (e) $\{(1,2,-2)^T, (2,3,-2)^T, (2,2,0)^T\}$

   Sometimes you'll be able to get from one part to the next by thinking, instead of computing!

2. Let $v_1, \cdots, v_m \in V$ be a linearly independent list of vectors. Suppose that $v \in V$. Prove that $\{v_1, \cdots, v_m, v\}$ is linearly independent if and only if $v \notin \text{span}(v_1, \cdots, v_m)$.

3. Let $V = \{f \in \mathcal{P}_3(\mathbb{R}) : p(1) = 0, p(-1) = 0\}$. You may use the fact that $\dim V = 2$.

   (a) Find two linearly independent elements in V.

   (b) Carefully prove that the elements you found in part (a) must span V.

4. Extend the set $\{1 + z + z^4, z + z^3\}$ to a basis of $\mathcal{P}_5(\mathbb{R})[z]$.

5. Suppose that $\text{span}(u_1, \cdots, u_m) = V$. Prove that there is some subset of the u_i which is a *basis* for V.