Homework 9

Due: Friday, April 19

In class, we described the symmetries of the square as

$$\mathrm{Sym}(\square) = \{I, R, R^2, R^3, H, HR, HR^2, HR^3\}.$$

1. There are two "diagonal reflection" elements in $\mathrm{Sym}(\square)$:

$$\boxed{\begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix}} \mapsto \boxed{\begin{matrix} 1 & 2 \\ 4 & 3 \end{matrix}}$$

$$\boxed{\begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix}} \mapsto \boxed{\begin{matrix} 3 & 4 \\ 2 & 1 \end{matrix}}$$

   Express each of them as $H^i R^j$ for suitable i and j.

2. Write out a complete Cayley table ("multiplication table") for $\mathrm{Sym}(\square)$.

3. If G is a group and $g \in G$, the order of g is the smallest positive number g such that

$$g^n = e.$$

   (Note: It may be that there is *no* such positive number, in which case we say g has infinite order.)

   Find the order of each element of $\mathrm{Sym}(\square)$.

4. If G is a group, its *center*, $Z(G)$, is

$$Z(G) = \{z \in G : \text{for each } g \in G, zg = gz\}.$$

   What is $Z(\mathrm{Sym}(\square))$?

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