Homework 7

Due: Friday, March 29

1. Let $\phi : R \to S$ be a ring homomorphism.
   (a) Suppose that ϕ is injective. Show that $\ker \phi = \{0\}$.
   (b) Suppose that $\ker \phi = \{0\}$. Show that ϕ is injective.

2. Suppose that $I \subset R$ is an ideal and $a \in I$. Show that $aR \subseteq I$. *The book uses the notation* $\langle a \rangle$ *for* aR.

3. [J]16.19.

4. Let F be a field. Show that every ideal in $F[x]$ is principal. (HINT: *The proof is a lot like our proof of the analogous fact for* $\mathbb{Z}$. *Let* I *be a nonzero ideal, and let* $f(x) \in I$ *be an element of smallest degree. Show that* $I = \langle f \rangle$.)

5. (a) Give an example of a zero divisor in $\mathbb{Z}/6\mathbb{Z}$.
   (b) Give an example of a zero divisor in $\mathbb{R}[x]/(x^2 - 1)$.

6. Let R be a commutative ring with identity. Let $I \subseteq R$ be an ideal. Suppose there are elements $a, b \in R$ such that $a \notin I$, $b \notin I$, but $ab \in I$.

   Show that R/I is not an integral domain. (HINT: *Abstract from your examples in* (5).)

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