Homework 6

Due: Friday, March 15

1. (a) Show, by explicitly computing all relevant sums and products, that the function

$$\begin{aligned} \mathbb{Z}_3 &\xrightarrow{\phi} \mathbb{Z}_6 \\ 0 &\longmapsto 0 \\ 1 &\longmapsto 4 \\ 2 &\longmapsto 2 \end{aligned}$$

   is a ring homomorphism.

   (b) Does this contradict Proposition 16.7.(3) of the text? Explain.

2. Let R be a ring. Recall that $0_R \in R$ is the unique element such that, for all $a \in R$, $a + 0_R = 0_R + a = a$.

   Suppose there are two elements $b, z \in R$ such that $b + z = b$. Show that $z = 0_R$.

3. Let $\phi : R \to S$ be a ring homomorphism. Show that $\phi(0_R) = 0_S$. (HINT: *Use* (2).)

4. Consider the ring homomorphism

$$\begin{aligned} \mathbb{R}[x] &\xrightarrow{\phi} \mathbb{R} \\ f(x) &\longmapsto f(0). \end{aligned}$$

   (a) Suppose that $f(x) = x \cdot g(x)$ for some polynomial $g(x) \in \mathbb{R}[x]$. Show that $f(x) \in \ker \phi$.

   (b) Conversely, suppose $f(x) \in \ker \phi$. Show that $f(x) = xq(x)$ for some polynomial $q(x)$. (HINT: *Divide $f(x)$ by x as in Theorem 17.4. What can the remainder be?*)