Homework 5

Due: Friday, March 1

1. Let R be a commutative ring with identity, and let $S \subset R$ be a ring which also contains the multiplicative identity element.

   (a) Suppose that R is an integral domain. Show that S is an integral domain.

   (b) If R is a field, must S be a field? Explain.

2. Let R be a ring with identity. Suppose that $a \in R$ is a zero divisor. Show that a is *not* a unit.

3. Consider the ring $\mathbb{Z}_n$, and suppose $0 < a < n$ satisfies $\gcd(a, n) = g > 1$. Show that $[a]$ is not a unit in $\mathbb{Z}_n$. (HINT: *Show that a is a zero divisor, and use 2.*)

4. (a) Compute $(5x^2 + 3x - 4)(4x^2 - x + 9)$ in $\mathbb{Z}_{12}[x]$.

   (b) Compute $(5x^2 + 3x - 4)(4x^2 - x + 9)$ in $\mathbb{Z}_{10}[x]$.

5. [J]17.3(ab).