Homework 4
Due: Friday, February 22

1. Consider the set of 2×2 matrices, $\mathrm{Mat}_2(\mathbb{R})$, equipped with the following binary operations:

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$$
$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \cdot \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{pmatrix}$$

   (a) Show that these operations turn $\mathrm{Mat}_2(\mathbb{R})$ into a ring.
   (b) What are the additive and multiplicative identity elements in this ring?
   (c) Which elements have a multiplicative inverse?

2. Consider the set of 2×2 matrices, $\mathrm{Mat}_2(\mathbb{R})$, equipped with the following binary operations:

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{pmatrix}$$
$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} * \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 & b_1b_2 \\ c_1c_2 & d_1d_2 \end{pmatrix}$$

   (a) Show that these operations turn $\mathrm{Mat}_2(\mathbb{R})$ into a ring.
   (b) What are the additive and multiplicative identity elements in this ring?
   (c) Which elements have a multiplicative inverse?

3. [J]16.1. *For each part, if the set* is *a ring, just say so. If it's* not *a ring, give a short example explaining why. Similarly, if the set is a ring but* not *a field, write down an element with no multiplicative inverse.*

4. [J]16.3(a)(b)(c).

5. Consider the ring of real polynomials $\mathbb{R}[x]$.

   (a) Prove that $\mathbb{R}[x]$ is an integral domain.
   (b) What are the units in $\mathbb{R}[x]$?

Bonus: Let R be a ring with additive identity 0_R and multiplicative identity 1_R. Show that if $0_R = 1_R$, then R only has a single element.