Homework 3

Due: Friday, February 15

1. [J]2.1.

2. (a) [J]2.5. (HINT: *If N is divisible by 3, then so is $N-3$.*)
   (b) Why does the same argument *not* show that each number $10^{n+1}+10^n+1$ is divisible by 9?

3. Here is another proof of the following result from class:

   Theorem Suppose $a \geq b > 0$. Then there are integers x and y such that $ax+by = \gcd(a,b)$.

   Actually, we don't need to assume a and b are positive; but it makes the writeup a little easier.

   Let
   $$S = \{am + bn : am + bn > 0, m \in \mathbb{Z}, n \in \mathbb{Z}\}.$$

   (a) Let d be the smallest element of S. Why does d exist?
   Henceforth, let $d = ax + by$.
   (b) Write $a = qd + r$ where $0 \leq r < d$. Show that in fact $r = 0$. (HINT: *Show that if $r > 0$, then $r \in S$. Why is this impossible?*)
   (c) Show that $d|a$ and $d|b$.
   (d) Suppose e is any divisor of a and b. Show that $e|d$. (HINT: *Write $a = eh$ and $b = ek$.*)

4. [J]2.15(a)(b)(c).

5. Suppose a is an integer. Show that $\gcd(a, a+1) = 1$.