
Monday, September 29

1. Recall (from day 1) the definition

$$M(n) = 2^n - 1.$$

Consider the statement

If n is prime, then $M(n)$ is prime.

- (a) What is the contrapositive of this statement?
 - (b) What is the converse of this statement?
 - (c) What is the contrapositive of the converse of this statement?
 - (d) Which of these statements are true?
2. An integer n can be written as

$$n = 4q + r$$

where q and r are integers and $r \in \{0, 1, 2, 3\}$. (We will say that $n \equiv r \pmod{4}$.)

Consider the statement:

If m is an integer such that $m \equiv 2 \pmod{4}$ or $m \equiv 3 \pmod{4}$, then m is not a perfect square.

- (a) What is the contrapositive of this statement?
- (b) Prove this contrapositive. (HINT: For each possible value of r , what is $(4q + r)^2 \pmod{4}$?)