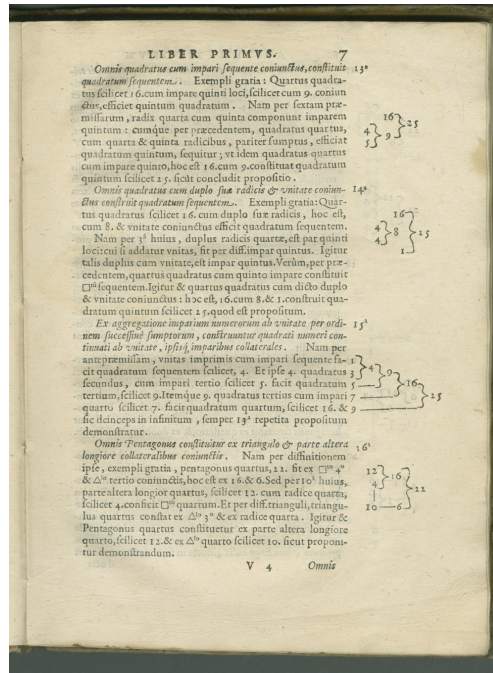1. In *Arithmeticorum libri duo* (1575), Maurolico shows that, for each positive integer n,

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

Prove his statement by induction.

2. Recall that the Fibonacci numbers F_n are defined by

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-2} + F_{n-1} \text{ if } n \geq 2.$$

Let

$$S_n = \sum_{j=1}^{n} F_j$$
$$= F_1 + F_2 + \cdots + F_n.$$

Prove that, for each n,

$$S_n = F_{n+2} - 1.$$

3. Prove by induction that, for each natural number n,

$$n < 2^n.$$